SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

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UNIT- I

Discrete Fourier Transform

Circular Convolution

Circular Convolution :

Def:

If $x(n)$ contains L no. of samples and $h(n)$ has M no. of samples such that $L > M$, then we perform circular convolution b/w the two sequences using $N = \max(L, M)$.

By adding $(L-M)$ no. of zero samples to the sequence $h(n)$.

There are two methods,

i) Concentric circle method
ii) Matrix method

i) Concentric circle method:

PROCEDURE:

Given $x_1(n)$ and $x_2(n)$ write the values of,

i) $x_1(n)$ → outer circle in counter (or) anti clockwise direction.

ii) $x_2(n)$ → inner circle in clockwise direction.

iii) Multiplying corresponding samples and sum the product to produce the output.

iv) Rotate the inner circle one sample at a time in counter clockwise direction and Repeat step iii).

PROBLEM :-

1) Find the circular convolution of two sequences,

$x_1(n) = \{1, 2, 2, 1\}$; $x_2(n) = \{1, 2, 3, 1\}$

using Concentric circle method & Matrix method.

Sol:

i) Concentric circle method:

$x_1(n) = \{1, 2, 2, 1\} \Rightarrow L = 4$

$x_2(n) = \{1, 2, 3, 1\} \Rightarrow M = 4$.

For circular convolution,

$y(n) = x_1(n) \textcircled{N} x_2(n)$.

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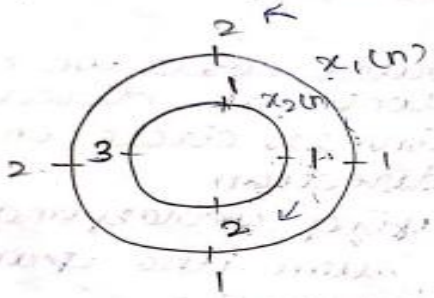
∴ Length of the output $y(n) \rightarrow N$

$N = \max(L, M)$

$N = \max(4, 4)$

$\boxed{N = 4}$

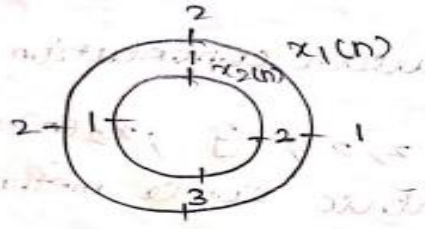
STEP. 1 :



$y(0) = 1+2+6+2$

$\boxed{y(0) = 11}$

STEP. 2 :



$y(1) = 2+3+2+2$

$\boxed{y(1) = 9}$

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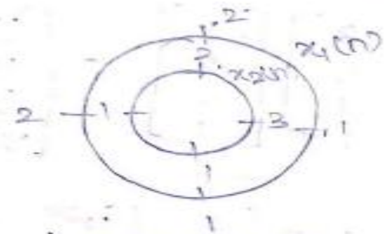
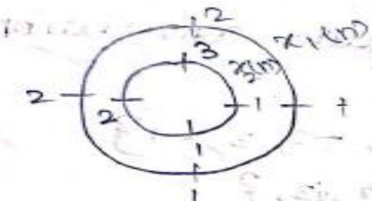
STEP 3:



$y(2) = 1 + 3 + 4 + 2$

$y(2) = 10$

STEP 4:



$y(3) = 1 + 1 + 6 + 4$

$y(3) = 12$

RESULT:

$y(n) = \{11, 9, 10, 12\}$

$(\because N = 4)$

ii) Matrix method:

$$\begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$x_1(n) = \{1, 2, 2, 1\}$

$x_2(n) = \{\underset{0}{1}, \underset{1}{2}, \underset{2}{3}, \underset{3}{1}\}$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$$y(n) = \begin{bmatrix} 1+2+6+2 \\ 2+2+2+3 \\ 3+4+2+1 \\ 1+6+4+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 10 \\ 12 \end{bmatrix}$$

Find the circular convolution of two sequences,

$x_1(n) = \{1, -1, -2, 3, -1\}$

$x_2(n) = \{1, 2, 3\}$

using concentric circle method & matrix method.

Soln: $y(n) = \{8, -2, -1, -4, -1\}$.

$x_1(n) = \{1, -1, -2, 3, -1\} \Rightarrow L = 5$

$x_2(n) = \{1, 2, 3\} \Rightarrow M = 3$.

$\Rightarrow (L-M)$

$= (5-3) = 2$ zeros.

$x_2(n) = \{1, 2, 3, 0, 0\}$

For circular convolution,

$y(n) = x_1(n) \textcircled{N} x_2(n)$.

∴ length of the o/p $y(n) \rightarrow N$.

$N = \max(L, M)$

$N = \max(5, 3)$

$\boxed{N = 5}$

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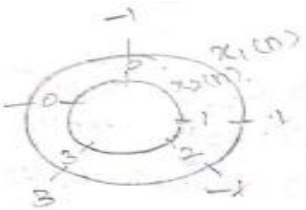
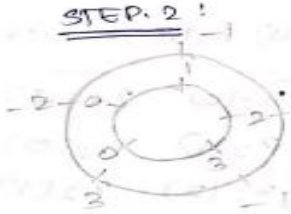
STEP. 1 :



$y(0) = 0+0+9-2+1$

$\boxed{y(0) = 8}$

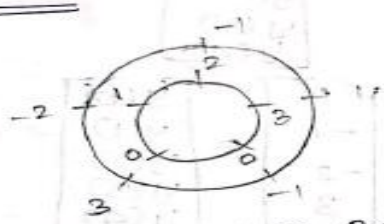
STEP. 2 :



$y(1) = 0+0-1-3+2$

$\boxed{y(1) = -2}$

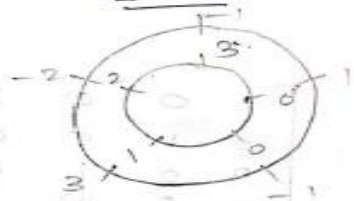
STEP. 3 :



$y(2) = 0+0+3-2-2$

$\boxed{y(2) = -1}$

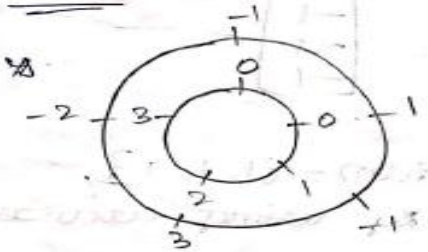
STEP. 4 :



$y(3) = 3+0+0-3-4$

$\boxed{y(3) = -4}$

STEP. 5 :



$y(4) = 6-1+0+0-6$

$\boxed{y(4) = -1}$

RESULT :

$y(n) = \{8, -2, -1, -4, -1\}$

$\therefore N = 5$

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Thank You!