SNS COLLEGE OF ENGINEERING

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UNIT- I

Discrete Fourier Transform

FFT Algorithms

FFT ALGORITHM :

(Fast Fourier Transform) :

It is a highly efficient algorithm for computing DFT of a finite sequence.

It requires less no. of computation when that required by direct evaluation of DFT.

It reduces computational complexity.

Applications :

1. Digital spectral analysis
2. Filter simulation
3. Auto correlation
4. Pattern recognition

Principles of FFT :

i) Breaking the transform into smaller transform and combining them to get the total transform.

ii) Twiddle factor :

It represents the complex no.,

$$W_N = e^{\frac{-j2\pi}{N}} \quad (or) \quad W_N^{nk} = e^{\frac{-j2\pi nk}{N}}$$

It helps in reducing the computational complexity.

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Properties of Twiddle factor :

i) Symmetry property :

$$W_N^{k+\frac{N}{2}} = -W_N^k$$

ii) Periodicity property :

$$W_N^{k+N} = W_N^k$$

iii) Direct computation of DFT :

the no. of complex multiplication required is $= N^2$.

the no. of complex addition required $= N(N-1)$.

iv) FFT :

the no. of complex multiplication required $= \frac{N}{2}\log_2 N$

the no. of complex addition required $= N \log_2 N$.

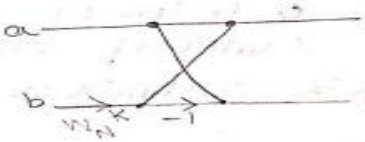
There are two classes of FFT algorithm.

i) Decimation in Time algorithm (DIT)

ii) Decimation in frequency algorithm (DIF)

Radix - 2

v) DIT : (Basic computational Blocks) :



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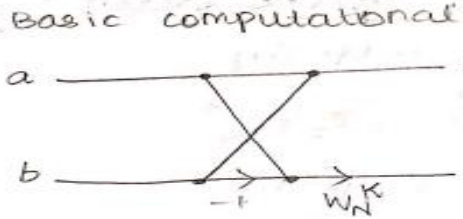
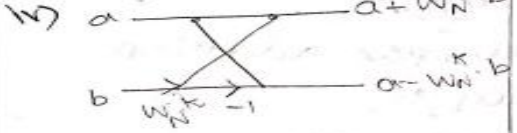
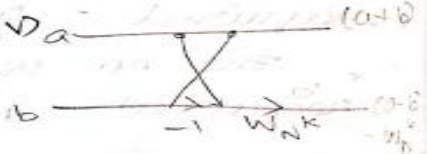
(or) ii) DIF (Basic computational block)



DIT	DIF
i) input is in bit reversed order.	i) input is in normal order
ii) output is in normal order.	ii) output is in bit reversed order
iii) Multiplication takes place before addition & subtraction operation	iii) multiplication takes place after addition & subtraction operation
iv) 	iv) 

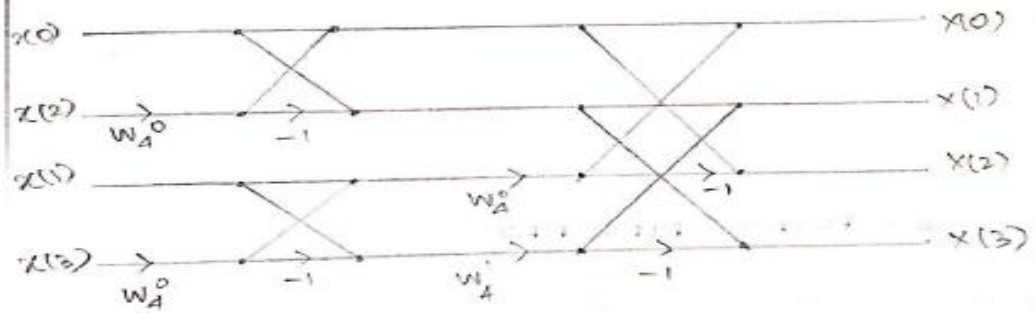
Define bit reversal:

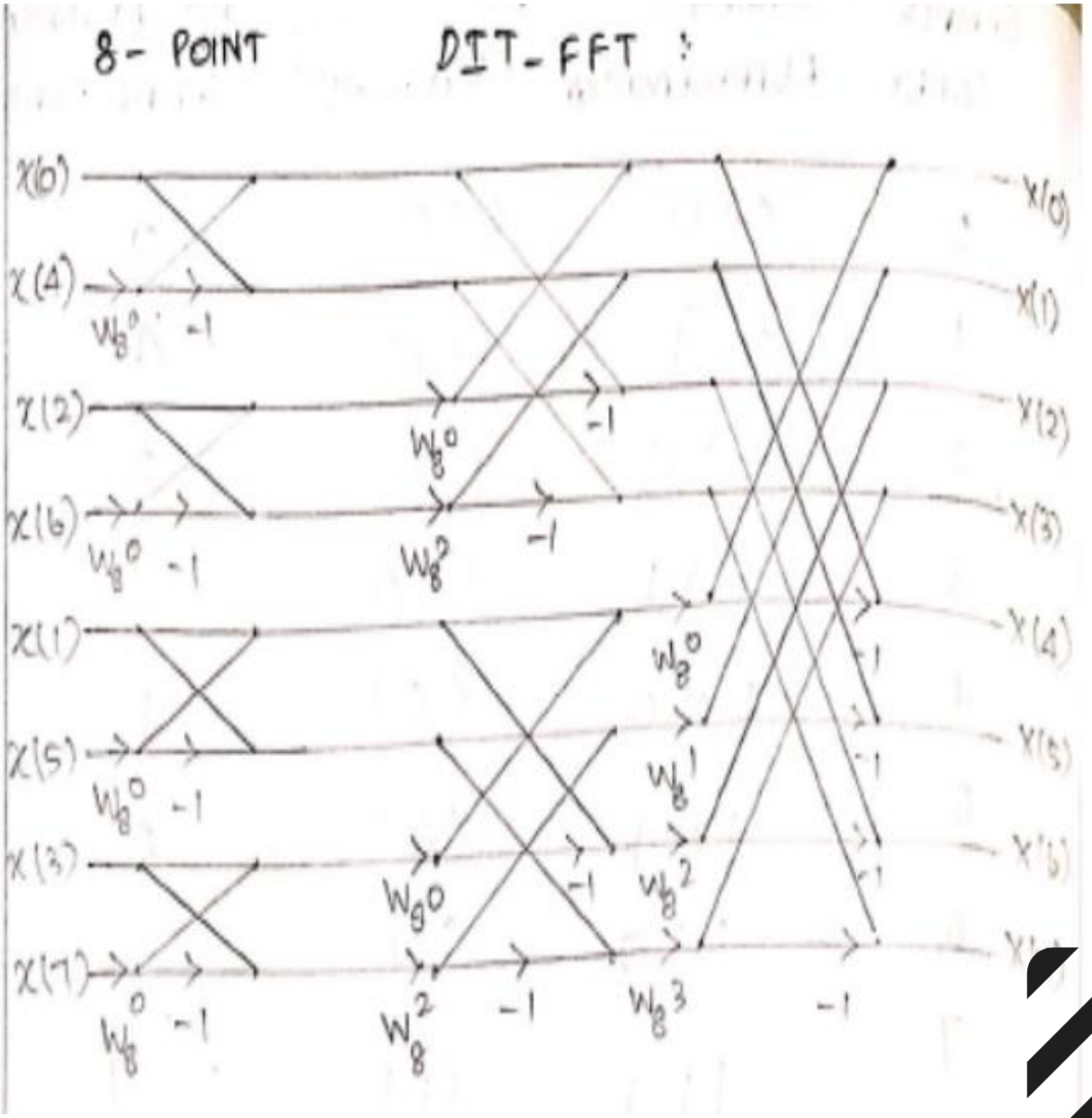
It is defined as, indexing the elements of the sequence by their nos. from 0 to $(n-1)$ and then reversing the binary representation of each of these numbers.

SAMPLE INDEX	BINARY REPRESENTATION	BIT REVERSAL	BIT REVERSAL SAMPLE INDEX
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

GENERAL STRUCTURE FOR DIT AND DIF ALGIORITHM:

4-point DIT, FFT – (Radix 4 Butterfly structure)




8- POINT DIT-FFT :
x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)
-1

4-POINT TWIDDLE FACTOR :

$$W_N^n = e^{\frac{-j2\pi n}{N}}$$

$$W_4^0 = e^{\frac{-j2\pi(0)}{4}} = 1$$

$$W_4^1 = e^{\frac{-j2\pi(1)}{4}} = e^{-j\pi/2} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= 0 - j(1)$$

$$\boxed{W_4^1 = -j}$$

8-POINT TWIDDLE FACTOR :

$$W_8^0 = e^{\frac{-j2\pi(0)}{8}} = e^0 = 1.$$

$$W_8^1 = e^{\frac{-j2\pi(1)}{8}} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4}$$

$$= 0.707 - j\,0.707$$

$$= 0.707(1-j)$$

$$W_8^2 = e^{\frac{-j2\pi(2)}{8}} = e^{\frac{-j4\pi}{8}2} = e^{-j\pi/2}$$

$$= \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= 0 - j(1)$$

$$= -j$$

$$W_8^3 = e^{\frac{-j2\pi(3)}{8}} = e^{\frac{-j6\pi}{8}4} = \cos\frac{3\pi}{4} - j\sin 3\pi/4$$

$$= -0.707 - j\,0.707$$

$$= 0.707(-1-j)$$

PROBLEM :

Compute DFT of the sequence $x(n) = \{1, 0, -1, 0\}$ using DIT – FFT algorithm.

SOL :

$x(n) = \{1, 0, -1, 0\}$

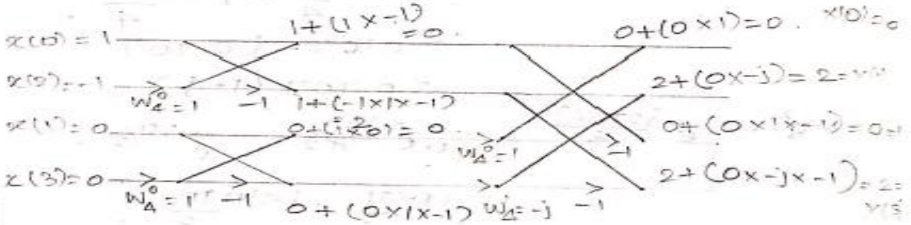
Here, $N = 4$.

$W_4^0 = 1$, $W_4^1 = e^{\frac{-j2\pi(1)}{4}}$

$= \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$

$= 0 - j(1)$

$= -j$.

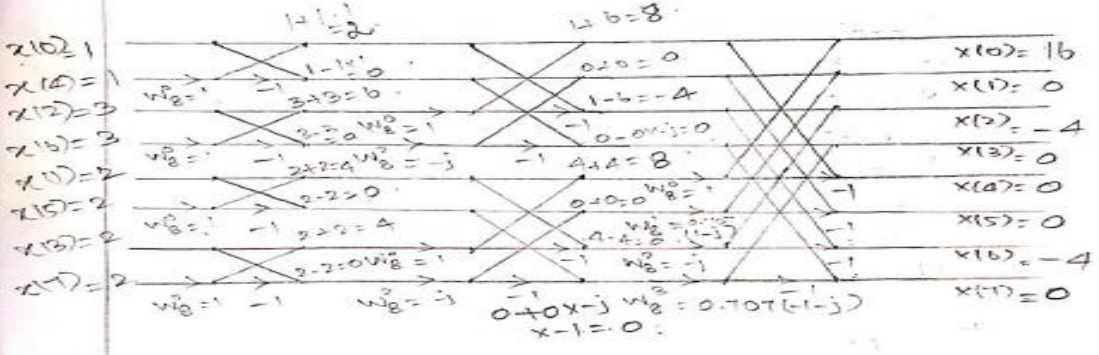


$X(K) = \{0, 2, 0, 2\}_K$

3) $x(n) = \{1, 2, 3, 2, 1, 2, 3, 2\}$

Sol:

DIT: $N = 8$



STAGE III

$x(0) = 8 + 8 \times 1 = 16$

$x(1) = 0 + (0 \times 0.707(1-j)) = 0$

$x(2) = -4 \times 0 \times -j = -4$

$x(3) = 0 + 0 \times 0.707(-1-j) = 0$

$x(4) = 8 + 8 \times -1 = 0$

$x(5) = 0 + 0 \times -1 = 0$

$x(6) = -4 + [0 \times -1] \times -j = -4$

$x(7) = 0 + 0 \times 0.707 \quad (-1-j) \times -1 = 0$

ANSWER:

$x(k) = \{16, 0, -4, 0, 0, 0, -4, 0\}$

Thank You!