TOPIC : 1.1 – Probability & Axioms of Probability

Probability & Random Variables :-

Introduction :-

In Ordinary language the word probability means uncertainity about happenings. Consider a day-to-day life statements :-

1) Every day the sun rises in the east.

2) It is possible to live without water

3) probably Arun gets that Job

look at the above statement there is certainity in the first, Impossibility in the second. And uncertainity in 3 statement.

In the theory of probability we represent Certainity by 1, Impossibility by 0 and uncertainity by positive fraction $(0 < \overset{b/w}{to} < 1) \Rightarrow 0 \leq 1$

Note :-

The terms probably, Chances, ~~lightly~~ likely, possible Convey the same meaning.

Deterministic Experiments

There are experiments which always produce the same result (or) unique outcome on every trial are called Deterministic Experiment

Eg) Throwing biased coin.

Random:

There are experiments which does not produce the same result (or) unique outcome on every trial

Eg) Throwing unbiased coin

Trial & event :-

The performance of a Random experiment is called a trial and outcome is called an event

Eg) Throwing of a coin is a trial and getting head (or) tail is an event

Sample space:-

The set of all possible outcomes of a Random experiment is called sample space.

The No. of sample points in a sample space is denoted as $n(S)$

Eg) Tossing a coin

$S = \{H, T\}$; $n(S) = 2$

2) Tossing a coin simultaneously

$S = \{HH, HT, TH, TT\}$; $n(S) = 4$

Exhaustive Events:

Events are said to be exhaustive when they include all possibilities

i) When tossing a coin either a head (or) tail turns up there is no other possibility ∴ They are Exhaustive events.

Equally likely events:

Events are called Equally likely when none of them can be performed rather than other.

Eg. When a coin is thrown as head is as likely turned up as tail. Hence H(T) are Equally likely events.

Mutually Exclusive events.

Events are called Mutually exclusive when no two of them can occur simultaneously.

Eg. If a coin is tossed either head can be up (or) tail can be up but both cannot be up at the same time.

Favourable event.

The trial which entail the happening of an event is said to be favourable of the event.

Independent events:

Events are said to be Independent with occurance of one will not depend on other.

Eg) If a coin is tossed twice the result of the second throw would in no way be affected by the result of I throw.

Dependent events:

If occurance of 1 event is affected by the occurance of other then the 2 event is depend on 1^{st}

Eg) If a person ~~dri~~ draw a Card from full pack & does not replace it. The result of the draw made afterwards will be depend on the 1^{st} draw

Probability :-

If a trial results "n" exhaustive mutually Exclusive and equally likely Cases and "m" of them are favourable to the happening of an event "E". Then the probablity p of happening is given by $P(E) = \frac{\text{No. of favourable Cases}}{\text{Total No. of exhaustive Cases}}$

$= \cancel{M/N}\ m/n$

Probability of Non happening of event

$\bar{E} = 1 - \frac{m}{n}$

$P(\bar{E}) = 1 - (P(E))$

Axioms of Probability:

Let S be a Sample Space A be a event associate with a random variable. then the probability of event A is denoted by $P(A)$ is defined as real number satisfies the following

1) $0 \le P(A) \le 1$

2) $P(S) = 1$

3) If $A_1, A_2 \ldots A_n$ are mutually exclusive event $P(A_1 \cup A_2 \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots P(A_n)$

(or) $P\left[\bigcup_{i=1}^{\infty} A_i\right] = \sum_{i=1}^{\infty} P(A_i)$

Theorem 1

The probability of an impossible event is 0

(i) $P(\phi) = 0$

Proof:

The sample space is a certain event

certain ← S = Sure event.

S & ϕ ⇒ Mutually exclusive

$$P(S \cup \phi) = P(S) + P(\phi)$$

but $S \cup \phi = S$

$$P(S) = P(S) + P(\phi)$$

$$P(\phi) = 0$$

Thm 2 :

If $\bar{A}$ (or) $A^c \Rightarrow$ Complement of A

$\bar{A}$ is the Complementary event of A

$$p(\bar{A}) = 1 - p(A)$$

Proof: A & $\bar{A}$ are mutually exclusive

$$p(A \cup \bar{A}) = p(A) + p(\bar{A})$$

But $A \cup \bar{A} = S$

$$p(S) = p(A) + p(\bar{A})$$

$$1 = p(A) + p(\bar{A})$$

$$\Rightarrow p(\bar{A}) = 1 - p(A)$$

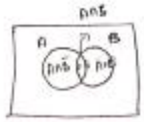
Addition theorem for Mutually Exclusive event.

If 2 events A & B are mutually Exclusive then $p(A \cup B) = p(A) + p(B)$

Addition thm for Non mutually Exclusive event

If A & B are any 2 events and are not disjoint then $p(A) + p(B) - p(A \cap B)$

Proof



$A \cup B$ is the union of 2 mutually exclusive events

$\bar{A} \cap B$ & A

$$A \cup B = A \cup (\bar{A} \cap B)$$

$$p(A \cup B) = p(A) + p(\bar{A} \cap B) \Rightarrow p(\bar{A} \cap B) = p(B) - p(A \cap B)$$

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$