TOPIC : 1.11 – Problems on Distribution

·Th time (in hours) required to repair a ma
ex ntially distributed with parameter $\lambda = \frac{1}{2}$.
what the prob. that a repair takes (i) atleast 10 h
Given the duration exceeds $\cancel{10}$ 9 hrs. (ii) the repair takes a time exceeds

L X denote the repair time of a machine.

Given is exponential with $\lambda = \frac{1}{2}$

of X $f(x) = \lambda e^{-\lambda x}, \; x \geq 0$

$$= \frac{1}{2} e^{-x/2}, \; x \geq 0$$

$$P[X > 10 / X > 9] = P[X > 9+1 / X > 9]$$

$$= P[X > 1] \quad \text{by memoryless property}$$

$$= e^{-\lambda(1)} = e^{-1/2} \quad [P(X>t) = e^{-\lambda t}]$$

$$= 0.6065$$

$$\text{(ii) } P[X > 2] = e^{-\lambda(2)} = e^{-1/2(2)} = e^{-1} = 0.3679$$

5) The mileage which car owners get with certain kind of radial tyre is a random variable having an exponential distribution with mean 4000 km. Find the prob. that one of these tyres will last
(i) atleast 2000 km (ii) atmost 3000 km.

Let the random variable x denote the mileage gn by a radial tyre.

Given x is uniformly exponential and mean = 4000

$\Rightarrow \frac{1}{\lambda} = 4000 \Rightarrow \lambda = \frac{1}{4000}$

$\therefore$ pdf of x $f(x) = \lambda e^{-\lambda x} = \frac{1}{4000} e^{-\frac{x}{4000}}, \quad x \geq 0$

(i) $P[x \geq 2000] = \int_{2000}^{\infty} P[\ e^{-\lambda(2000)} = e^{-\frac{1}{2}} = 0.6065$

(ii) $P(x \leq 3000) = 1 - P(x > 3000) = 1 - e^{-\frac{1}{4000} \times 3000} =$

$= 1 - e^{-\frac{3}{4}} = 0.5276$

4) If a continuous random variable x follows distribution in the interval $(0,2)$ & continuous random variable Y follows exponential distribution with par α, find α such that $P[x<1] = P[Y<1]$

Given X is uniformly distributed over $(0,2)$

$\therefore$ pdf of X $f(x) = \frac{1}{b-a} = \frac{1}{2}, \quad 0 < x < 2$

and Y is exponentially distributed with parameter

$\therefore$ pdf of Y $f(y) = \alpha e^{-\alpha y}, \quad y \geq 0$

Given that $P(x<1) = P(Y<1)$

$\Rightarrow \int_0^1 f(x)\,dx = \int_0^1 \alpha e^{-\alpha y}\,dy$

$\Rightarrow \frac{1}{2}(x)_0^1 = \alpha \left(\frac{e^{-\alpha y}}{-\alpha}\right)_0^1$

$\Rightarrow \frac{1}{2} = -(e^{-\alpha} - 1) \Rightarrow e^{-\alpha} - 1 = -\frac{1}{2}$

$\Rightarrow e^{-\alpha} = \frac{1}{2}$

$\Rightarrow e^{\alpha} = 2 \Rightarrow \boxed{\alpha = \log_e 2}$

If X is uniformly distributed with r… and variance $\frac{4}{3}$. Find $P(X<0)$

Given Mean $= \frac{a+b}{2} = 1 \Rightarrow a+b = 2$ —

Variance $= \frac{(b-a)^2}{12} = \frac{4}{3}$

$\Rightarrow (b-a)^2 = 16$

$\Rightarrow b-a = \pm 4 \longrightarrow (2)$

Take $b-a = 4$

$b+a = 2$

$2b = 6$

$\boxed{b = 3}$

$\Rightarrow \boxed{a = -1}$

Take $b-a = -4$

$b+a = 2$

$2b = -2$

$b = -1$

$b+a = 2$

$-1+a = 2$

$\boxed{a = 3}$

which contradicts a

$\therefore a = -1$ and $b = 3$

pdf of X $f(x) = \frac{1}{b-a} = \frac{1}{3-(-1)} = \frac{1}{4}$

$P(X<0) = \int_{-1}^{0} \left(\frac{1}{4}\right) dx = \frac{1}{4}(x)_{-1}^{0} = \frac{1}{4}$

A random variable X has a uniform distribu[tion]
[ove]r $(-3,3)$. Compute (i) $P(X<2)$ (ii) $P(|X|<2)$,
[(iii)] $P(|X-2|<2)$ (*)

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Given X is uniform distribution over $(-3,3)$.

$\therefore$ Pdf of X $f(x) = \frac{1}{b-a} = \frac{1}{6}$, $-3<x<3$.

(i) $P(X<2) = \int_{-3}^{2} f(x)\,dx = \int_{-3}^{2} \frac{1}{6}\,dx$

$= \frac{1}{6}(x)_{-3}^{2} = \frac{5}{6}$

(ii) $P(|X|<2) = P(-2<X<2) = \int_{-2}^{2} f(x)\,dx$

$= \int_{-2}^{2} \frac{1}{6}\,dx = \frac{1}{6}(x)_{-2}^{2} = \frac{4}{6} = \frac{2}{3}$

(iii) $P(|X-2|<2) = P(-2<X-2<2)$

$= P(0<X<4)$

$= \int_{0}^{3} f(x)\,dx = \int_{0}^{3} \frac{1}{6}\,dx = \frac{1}{6}(x)_{0}^{3}$

$= \frac{3}{6} = \frac{1}{2}$