Motion of follower

The follower, during its travel, may have one of the following motions:

a Uniform velocity
b Simple harmonic motion
c Uniform acceleration and retardation
d Cycloidal motion

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. 4.4 (a), (b) and (c) respectively.

The abscissa (base) represents the time (i.e. the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower.

Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB1 and C1D must be straight lines.

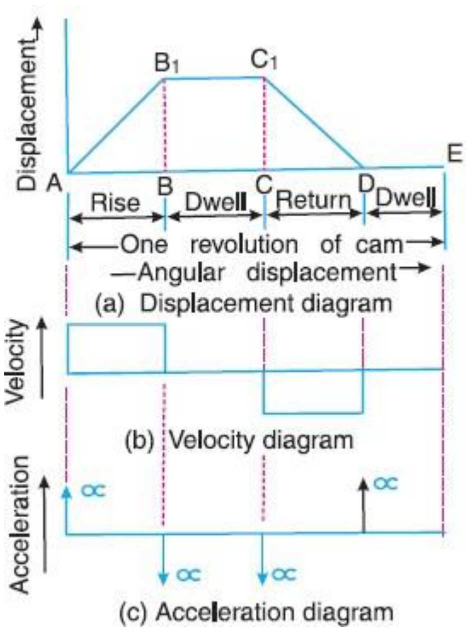


Fig. 4.4 displacement, velocity and acceleration diagrams

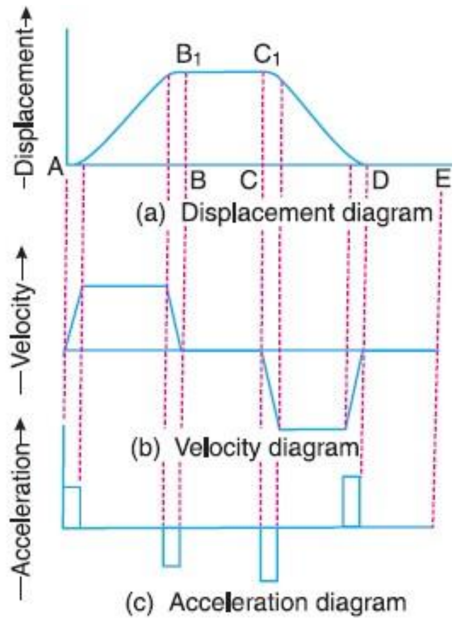


Fig. 4.5 modified displacement, velocity acceleration diagrams

A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as dwell periods, as shown by lines B1C1 and DE in Fig. 4.4 (a). From Fig. 4.4 (c), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.

In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke, as shown in Fig. 4.5 (a). By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 4.5 (b).

The modified displacement, velocity and acceleration diagrams are shown in Fig.4.5. The round corners of the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 4.6 (a), (b) and (c) respectively. The displacement diagram is drawn as follows:

a Draw a semi-circle on the follower stroke as diameter.

b Divide the semi-circle into any number of even equal parts (say eight).

c Divide the angular displacements of the cam during out stroke and return stroke into the
same number of equal parts.

d The displacement diagram is obtained by projecting the points as shown in Fig. 7.6 (a).

The velocity and acceleration diagrams are shown in Fig. 4.6 (b) and (c) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve.

We see from Fig. 4.6 (b) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.

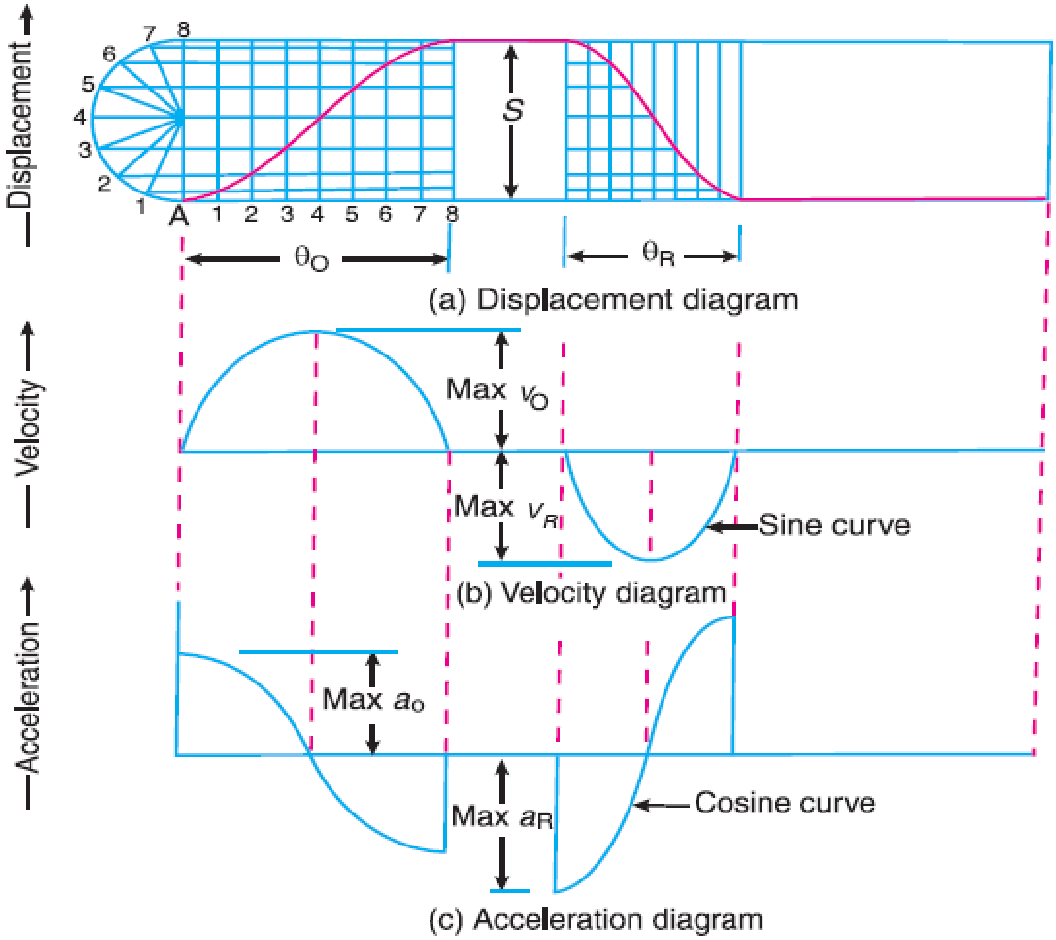


Fig. 7.6 acceleration diagram

4.3.1 L

Se= Stroke of the follower

t

,

Θ_0 and Θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively

ω = angular velocity of cam

Time required for the outstroke of the follower in second

$$t_0 = \frac{0}{\omega}$$

Consider a point P moving at uniform speed ω_p radians per sec round the circumference of a circle with the stroke S as diameter, as shown in Fig. 7.7 the point (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

Peripheral speed of the point P'

$$v_p = \frac{\pi \times s}{2} \times \frac{1}{t_0} = \frac{\pi \times s}{2} \times \frac{\omega}{\theta_0}$$

and maximum velocity of the follower on the outstroke,

$$v_0 = v_p = \frac{\pi \times s}{2} \times \frac{\omega}{\theta_0} = \frac{\pi \times \omega \times s}{2\,\theta_0}$$

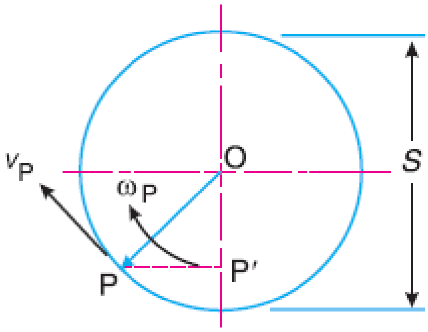


Fig. 7.7 motion of a point

We know that the centripetal acceleration of the point P

$$a_p = \frac{{v_p}^2}{op} = \left(\frac{\times\ \omega\ \times s}{2\ \theta_0}\right)^2 \times \frac{2}{s} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta_0)^2}$$

Maximum acceleration of the follower on the outstroke,

$$a_0 = a_p = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta_0)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \times \omega \times S}{2\ \theta_R}$$

and maximum acceleration of the follower on the return stroke

$$a_R = \frac{\pi^2 \omega^2 S}{2\ (\theta_R)^2}$$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 4.8 (a), (b) and (c) respectively. We see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below:

Divide the angular displacement of the cam during outstroke (ϴ) into any even number of equal parts and draw vertical lines through these points as shown in fig. 4.8 (a)

Divide the stroke of the follower (S) into the same number of equal even parts.

Join Aa to intersect the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. as shown in Fig. 20.8 (a). Now join these points to obtain the parabolic curve for the out stroke of the follower.

In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.

We know that time required for the follower during outstroke,

$$t_0 = \frac{0}{\omega}$$

and time required for the follower during return stroke,

$$t_R = \frac{\theta_R}{\omega}$$

Mean velocity of the follower during outstroke

$$v_0 = \frac{S}{t_0}$$

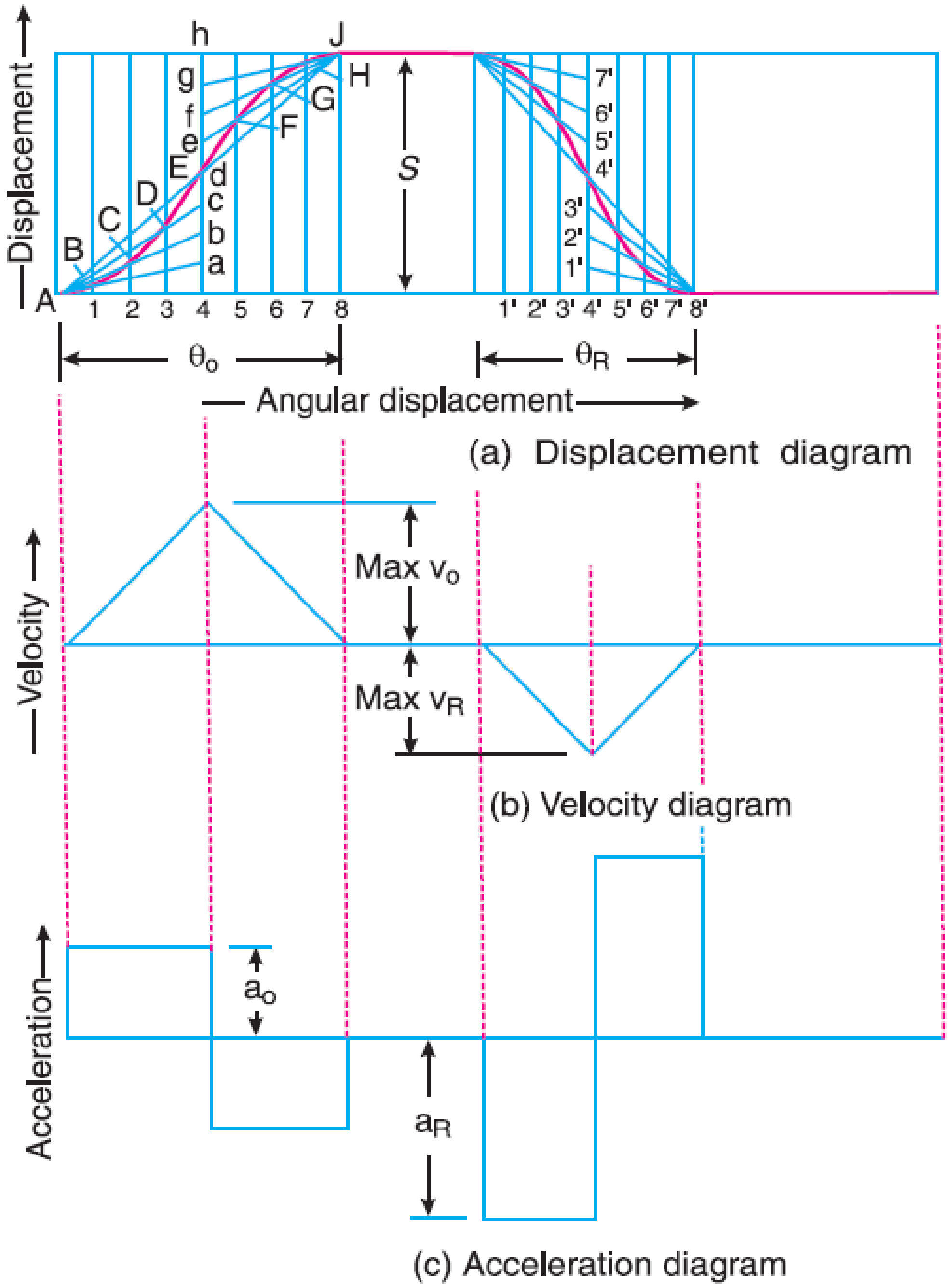


Fig. 4.8 Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

Since the maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_0 = \frac{2\,S}{t_0} = \frac{2\,\omega\,S}{\theta_0}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega\,S}{\theta_R}$$

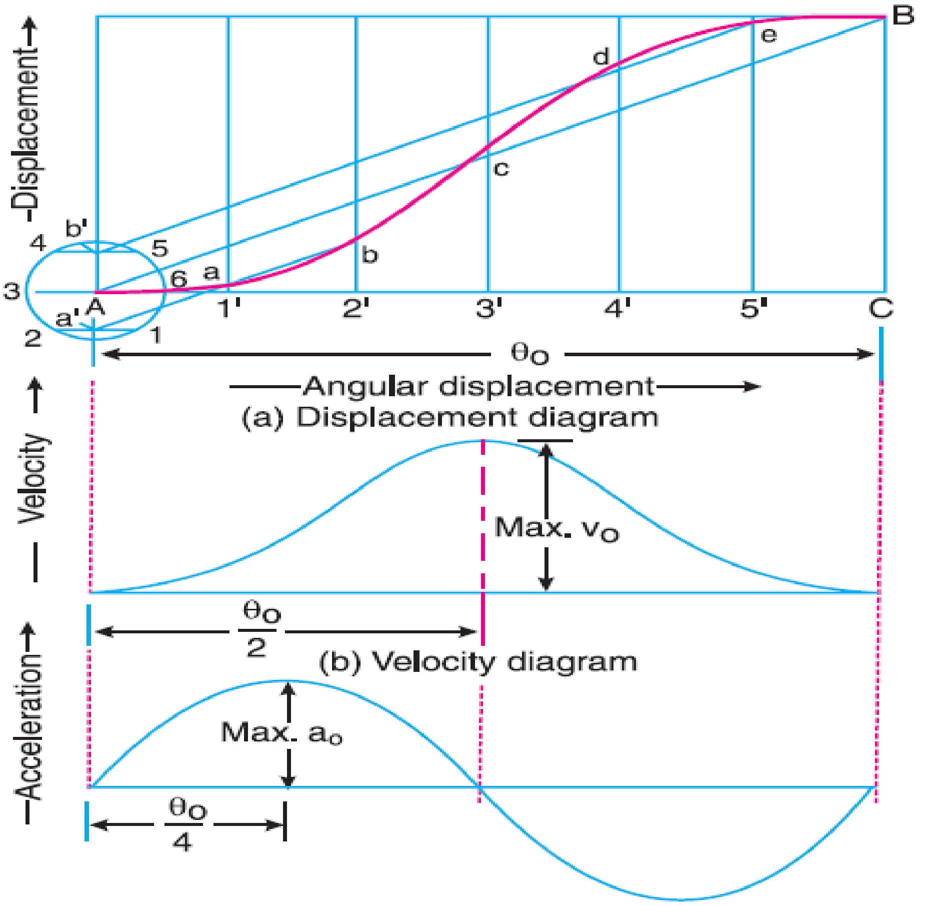
Maximum acceleration of the follower during outstroke,

$$a_0 = \frac{v_0}{t_0/2} = \frac{2 \times 2\,\omega\,s}{t_0\,\theta_0} = \frac{4\,\omega^2 . S}{(\,)^2{}_0}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2\,S}{(\theta\,)^2}$$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with cycloidal Motion



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

– The displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion are shown in Fig. (a), (b) and (c) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line.
– We know that displacement of the follower after time t seconds,

$$x = S\ [\frac{\theta}{\theta_0} - \frac{1}{2\,\pi}\ \sin(\frac{2\,\pi\,\theta}{\theta_0})]$$

– Velocity of the follower after time t seconds,

$$\frac{dx}{dt} = S\ [\frac{1}{\theta_0} \times \frac{d\theta}{d} - \frac{2\,\pi\,\theta}{\theta_0}\cos(\frac{2\,\pi\,\theta}{\theta_0})\ \frac{d\theta}{dt}]$$

$$= \frac{S}{\theta_0} \times \frac{t}{\frac{d\theta}{dt}}\ [1 - \cos(\frac{2\,\pi\,\theta}{\theta_0})]$$

$$= \frac{\omega\,S}{\theta_0}\ [1 - \cos(\frac{2\,\pi\,\theta}{\theta_0})]$$

– The velocity is maximum, when

$$\cos(\frac{2\,\pi\,\theta}{\theta_0}) = -1$$

$$\frac{2\,\pi\,\theta}{\theta_0} = \pi$$

$$= \frac{\theta_0}{2}$$

– Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega\,S}{\theta_R}$$

– Now, acceleration of the follower after time t sec,

$$\frac{d^2x}{dt^2} = \frac{\omega\,S}{\theta_0}\ \frac{2\,\pi}{\theta_0}\ \sin(\frac{2\,\pi\,\theta}{\theta_0})\ \frac{d\theta}{dt}]$$

$$= \frac{2\,\pi\,\omega^2\,S}{(\ )^2_0}\ \sin(\frac{2\,\pi\,\theta}{\theta_0})$$

– The acceleration is maximum, when

$$\sin(\frac{2\,\pi\,\theta}{\theta_0}) = 1$$

$$= \frac{\theta_0}{4}$$

$$a_0 = \frac{2\,\pi\,\omega^2\,S}{(\theta_0)^2}$$

$$a_R = \frac{2\,\pi\,\omega^2\,S}{(\theta)^2}$$

Construction of cam profile for a Radial cam

In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the opposite direction to the cam rotation.

Examples based on cam profile

Draw the profile of a cam operating a knife-edge follower having a lift of 30 mm. the cam raises the follower with SHM for 150º of the rotation followed by a period of dwell for 60º. The follower descends for the next 100º rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotates at a uniform velocity of 120 rpm and has a least radius of 20 mm. what will be the maximum velocity and acceleration of the follower during the lift and the return?

- S = 30 mm : Øa = 150º ; N = 120 rpm ;
- $\delta_1 = 60°$; $r_c = 20$ mm : $\delta_2 = 50°$
- **During ascent:**

$$\omega = \frac{2\,\pi\,N}{60} = \frac{2\,\pi \times 120}{60} = 12.57\ rad/s$$

$$v_{max} = \frac{\pi \times \omega \times s}{2\,\theta_0} = \frac{\pi \times 12.57 \times 30}{2 \times 150 \times \frac{\pi}{180}} = 226.3$$

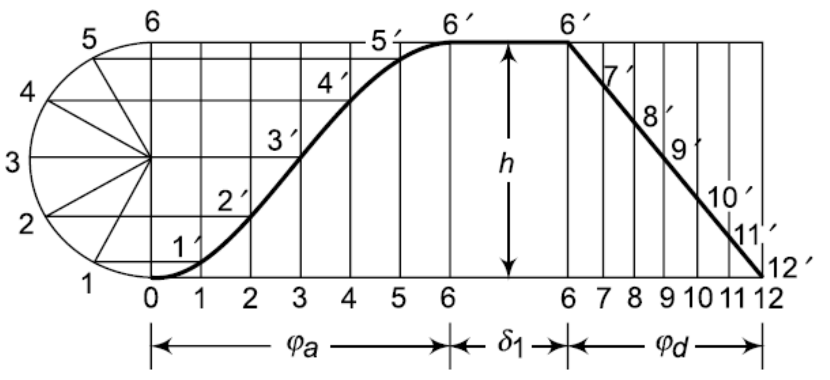
$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta^0)^2} = \frac{\pi^2 \times 12.57^2 \times 30}{2 \times (150 \times \frac{\pi}{180})^2} = 7.413\ {}^{m}/_{s^2}$$

- **During descent:**

$$v_{max} = \frac{\omega\,S}{\emptyset_d}$$

$$v_{max} = \frac{12.57 \times 30}{100 \times \frac{}{180}} = 216\ {}^{mm}/_{s}$$

$$\boldsymbol{f_{max} = 0}$$



(a)

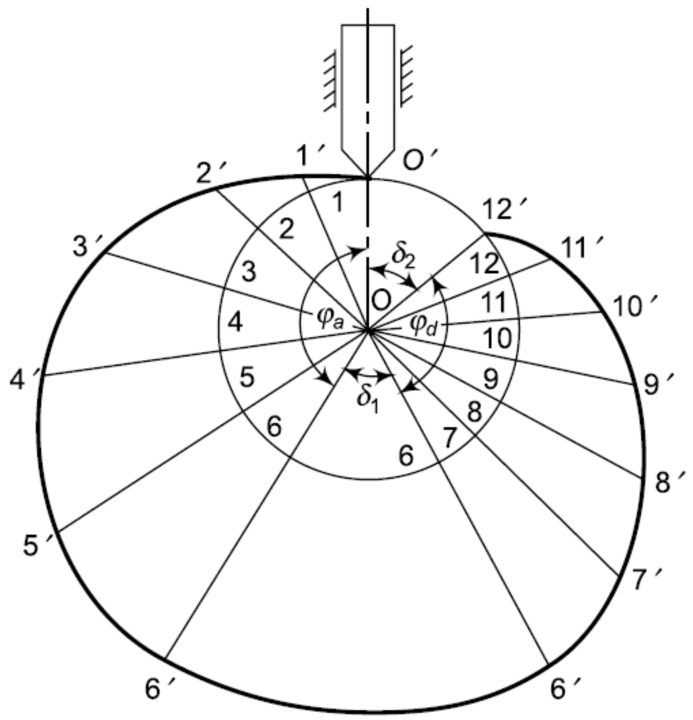


Fig. 4.10

A cam with a minimum radius of 25 mm is to be designed for a knife-edge follower with the following data:

To raise the follower through 35 mm during 60^o rotation of the cam

Dwell for next 40^o of the cam rotation

Descending of the follower during the next 90^o of the cam rotation

Dwell during the rest of the cam rotation

Draw the profile of cam if the ascending and descending of the cam with simple harmonic motion and the line of stroke of the follower is offset 10 mm from the axis of the cam shaft.

What is the maximum velocity and acceleration of the follower during the ascent and the descent if the cam rotates at 150 rpm?

- S = 35 mm : Øa = 60^o ; N = 150 rpm ;
- $\delta_1 = 40°$; r_c = 25 mm : $Ø_d$ = 90^o; x = 10 mm

– **During ascent:**

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 5\pi \ \frac{rad}{s}$$

$$v_{max} = \frac{\pi \times \omega \times s}{2\,\theta_0} = \frac{\pi \times 5\pi \times 35}{2 \times 150 \times \frac{\pi}{180}} = 827.7\ mm/s$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta_0)^2} = \frac{\pi^2 \times (5\pi)^2 \times 35}{2 \times (150 \times \frac{\pi}{180})^2} = 38.882\ m/s^2$$

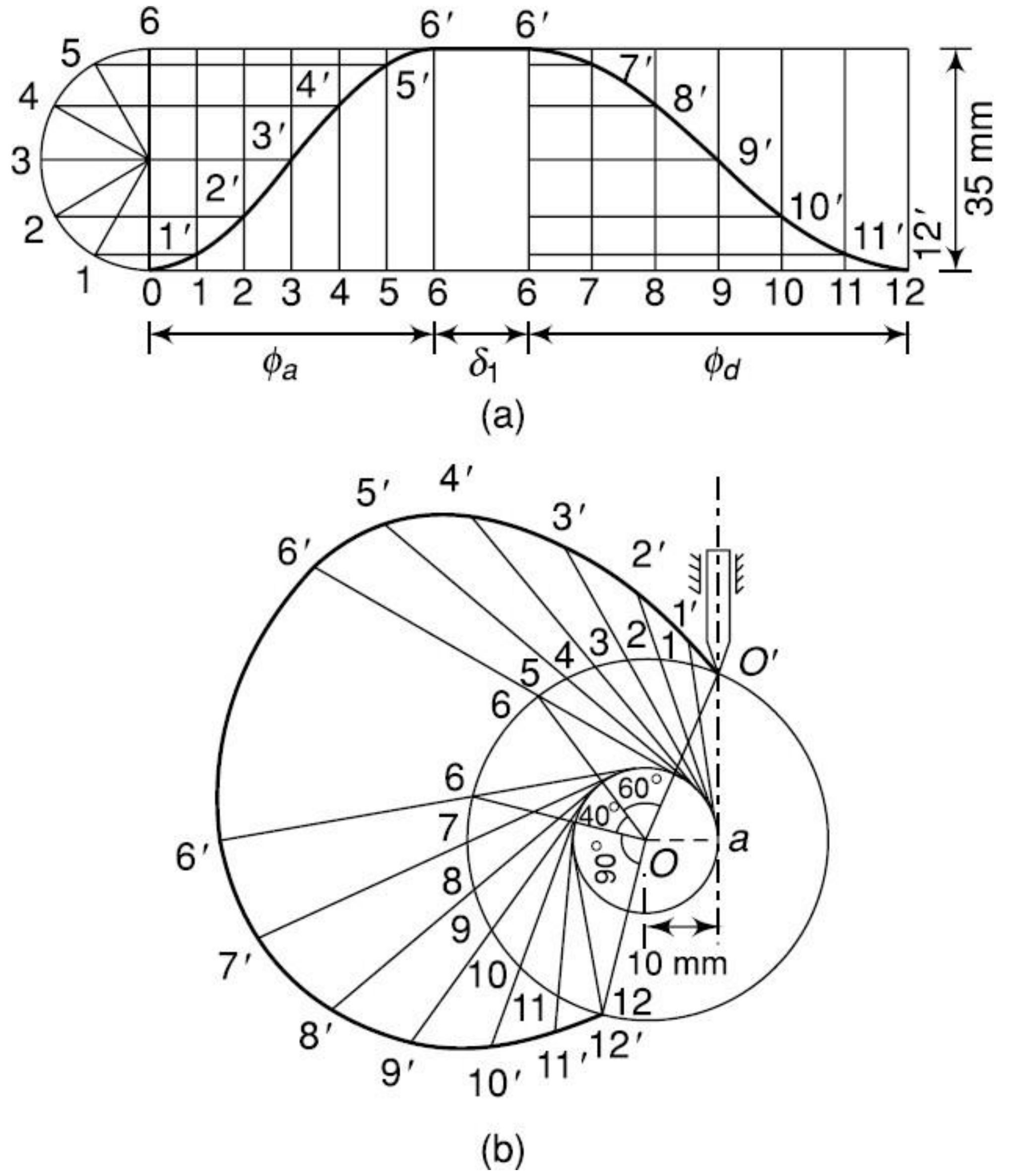


Fig. 7.11

– **During descent:**

$$v_{max} = \frac{\pi \times \omega \times s}{2\,\theta_0} = \frac{\pi \times 5\pi \times 35}{2 \times 90 \times \frac{\pi}{180}} = 549.80\ mm/s$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta^0)^2} = \frac{\pi^2 \times 5\pi^2 \times 35}{2 \times (90 \times \frac{\pi}{180})^2} = 17.272 \ m/s^2$$

A cam is to give the following motion to the knife-edged follower:

To raise the follower through 30 mm with uniform acceleration and deceleration during 120⁰ rotation of the cam

Dwell for the next 30⁰ of the cam rotation

To lower the follower with simple harmonic motion during the next 90⁰ rotation of the cam

Dwell for the rest of the cam rotation

The cam has minimum radius of 30 mm and rotates counter-clockwise at a uniform speed of 800 rpm. Draw the profile of the cam if the line of stroke of the follower passes through the axis of the cam shaft.

- S = 30 mm : Øa = 120⁰ ; N = 800 rpm ;
- $\delta_1 = 30°$; r_c = 30 mm : $Ø_d$ = 90⁰;
- **During ascent:**

$$\omega = \frac{2 \pi N}{60} = \frac{2 \pi \times 840}{60} = 88 \frac{rad}{s}$$

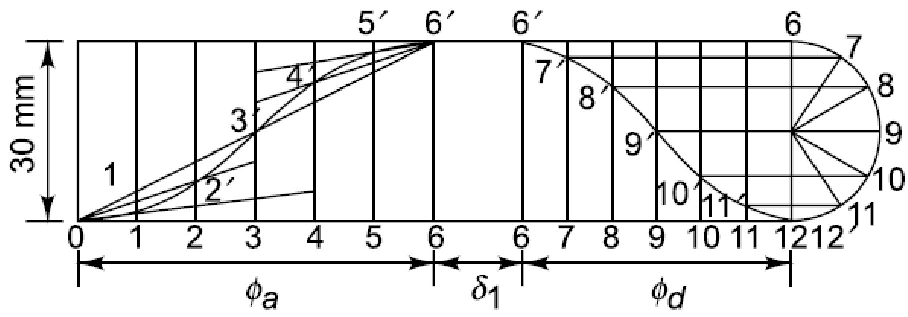
$$v_{max} = \frac{2 \times 88 \times 0.03}{120 \times \frac{\pi}{180}} = 2.52 \ m/s$$

$$a_0 = \frac{4 \omega^2 . S}{(\ )^2} = \frac{4\ 88^2 \times 0.03}{(120 \times \frac{\pi}{180})^2} = 211.9 \ m/s^2$$

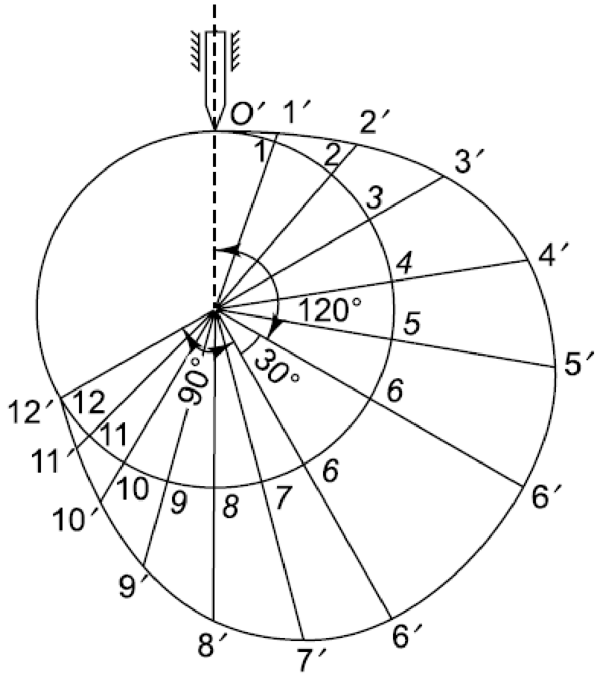
- **During descent:**

$$v_{max} = \frac{\pi \times \ \times s2}{\theta_0} = \frac{\pi \times 88 \times 0.03}{2 \times 90 \times \frac{\pi}{180}} = 2.64 \ mm/s$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta_0\ )^2} = \frac{\pi^2 \times 88^2 \times 0.03}{2 \times (90 \times \frac{\pi}{180})^2} = 467.6 \ m/s^2$$



(a)



(b)

Fig. 7.12

Draw the profile of a cam operating a roller reciprocating follower and with the following data:

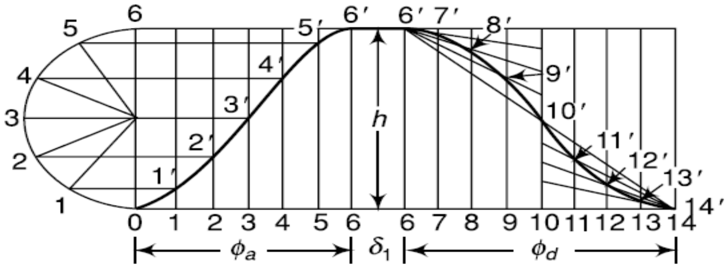
Minimum radius of cam = 25 mm

Lift = 30 mm

Roller diameter = 15 mm

The cam lifts the follower for 120⁰ with SHM followed by a dwell period of 30⁰. Then the follower lowers down during 150⁰ of the cam rotation with uniform acceleration and deceleration followed by dwell period. If the cam rotates at a uniform speed of 150 rpm. Calculate the maximum velocity and acceleration of the follower during the descent period.

- S = 30 mm : $Øa = 120^0$; N = 150 rpm ; $Ø_d = 150^0$
- $\delta_1 = 30°$; $r_c = 25$ mm : $\delta_2 = 60°$; $r_r = 7.5$ mm



(a)