TOPIC : 1.3 – Baye's theorem & Problems

Baye's theorem:

If $B_1, B_2 \dots B_n$ be a set of exhaustive and mutually exclusive events associated with random experiment and A is another event associate with B_i

Then
$$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$$

Proof:

$$P(B_i \cap A) = P(B_i) \cdot P(A/B_i)$$
$$= P(A) \cdot P(B_i/A)$$

$$P(B_i/A) = \frac{P(B_i \cap A)}{P(A)} \quad \text{[By conditional Probability]}$$

$$= \frac{P[B_i] \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)} \implies \text{By Total probability}$$

problem based on Baye's theorem:-

1) Suppose there are 3 urns containing 2 white 3 black, 3w, 2B & 4 w & 1 b respectively. There is equal probability of each urn being choosen. One ball is drawn from

an urn choosen at random. What is the prob that white ball is drawn from the 1st urn?

Soln

Let B_1 be the event that 1st urn choosen

Let B_2 be the event that 2nd urn choosen

Let B_3 be the event that 3rd urn choosen.

Let A be the event that a W ball is drawn.

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

$P(A|B_1) = \frac{2}{5}$; $P(A|B_2) = \frac{3}{5}$; $P(A|B_3) = \frac{4}{5}$

∴ By baye's thm probab of WB being drawn out of the 1st urn is given by

$$P(B_1|A) = \frac{P(B_1)P(A|B_1)}{\sum_{i=1}^{3} P(B_i)P(A|B_i)}$$

$$= \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{\frac{1}{3}\cdot\frac{2}{5}}{\frac{1}{3}\left[\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right]} = \frac{\frac{2}{5}}{9/5}$$

$= \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}.$

2) A bag contains 5 balls & it is not known how many of them are white. 2 Balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white.

Soln.

Since 2 W balls have drawn out, The bag must have contain 2, 3, 4 (or) 5 W balls.

Let B_1 event of bag containing 2 W Balls
B_2 " " 3 W Balls
B_3 " " 4 W balls
B_4 " " 5 W balls

Let A be the event of drawing 2 white balls.

Since no. of W balls in the bag is not known, B_i's are equally likely

$P(B_1) = P(B_2) = P(B_3) = P(B_4) = \frac{1}{4}$

$B_1 \Rightarrow 2W = P(B_1)$

$B_2 = 3W = P(B_2)$

$B_3 = 4W = P(B_3)$

$B_4 = 5W = P(B_4)$

$$P(B_4/A) = \frac{P(B_4) \cdot P(A/B_4)}{\sum_{i=1}^{4} P(B_i) \cdot P(A/B_i)}$$

$$P(A/B_1) = \frac{2c_2}{5c_2} = \frac{2\times 1}{1\times 2} = \frac{1}{10} \quad \frac{5\times 4}{1\times 2} = \frac{20}{2}$$

$$P(A/B_2) = \frac{3c_2}{5c_2} = \frac{3\times 2}{1\times 2} = \frac{6}{2} \quad \frac{3}{10}$$

$$P(A/B_3) = \frac{4c_2}{5c_2} = \frac{4\times 3\times 2}{1\times 2\times 3} = \frac{24}{6} = \frac{6}{10}$$

$$P(A/B_4) = \frac{5c_2}{5c_2} = 1$$

$$= \frac{\frac{1}{4} \cdot 1}{\frac{1}{4}\left[\frac{1}{10} + \frac{3}{10} + \frac{6}{10} + 1\right]} = \frac{1}{\left(\frac{20}{10}\right)} = \frac{10}{20} = \frac{1}{2}.$$