SNS COLLEGE OF ENGINEERING

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

INTRODUCTION TO BOOLEAN ALGEBRA

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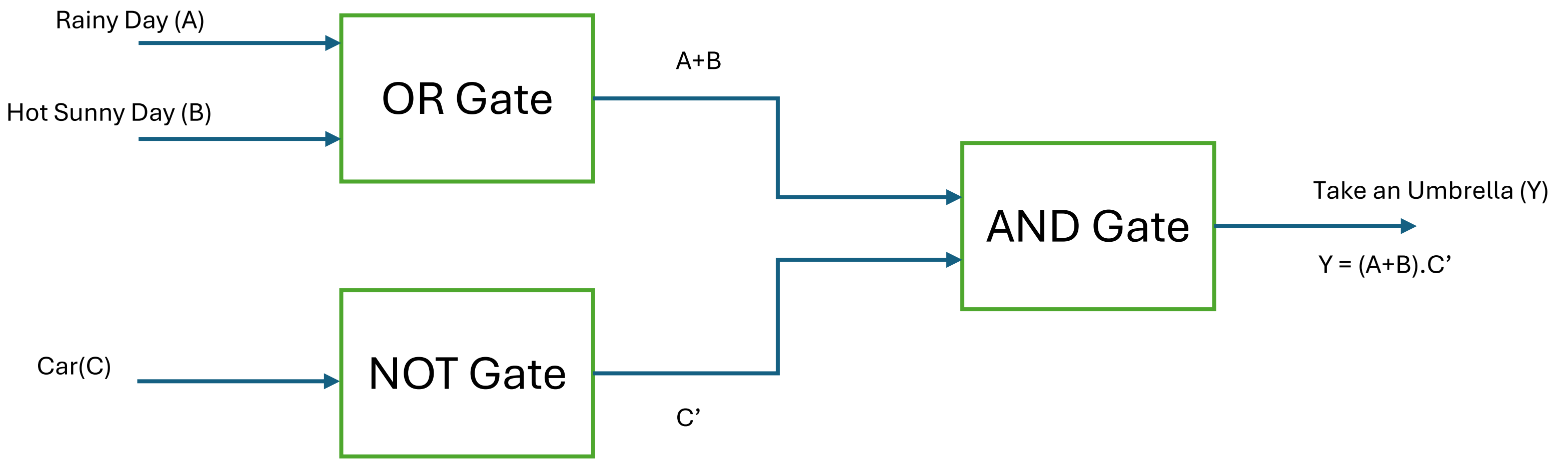
INTRODUCTION TO BOOLEAN ALGEBRA

- Boolean algebra was invented by **George Boole** in 1854.
- Boolean algebra is a branch of mathematics that deals with **operations on logical values** with binary variables.
- The Boolean variables are represented as **binary numbers** to represent truths: 1 = true and 0 = false.
- Boolean Algebra is used to analyze and simplify the **digital (logic) circuits.**

BOOLEAN ALGEBRA

- Consider an example



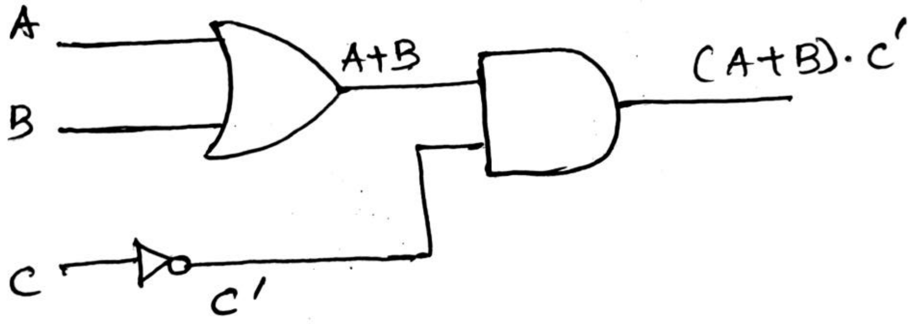
BOOLEAN EXPRESSION IS : $Y = (A+B) . C'$

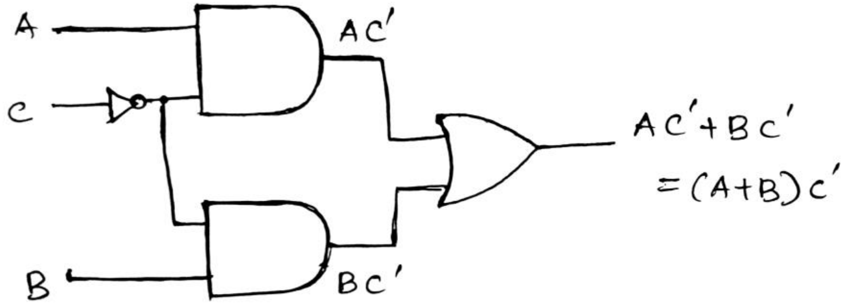
BOOLEAN EXPRESSION

BOOLEAN EXPRESSION IS : **Y = (A+B) . C'** ---------------Eqn (1)

Expanding,

BOOLEAN EXPRESSION IS : **Y = AC'+BC'** ---------------Eqn (2)





BOOLEAN LAWS

Law/Theorem	Law of Addition	Law of Multiplication
Identity Law	$x + 0 = x$	$x \cdot 1 = x$
Complement Law	$x + x' = 1$	$x \cdot x' = 0$
Idempotent Law	$x + x = x$	$x \cdot x = x$
Dominant Law	$x + 1 = 1$	$x \cdot 0 = 0$
Involution Law	$(x')' = x$	
Commutative Law	$x + y = y + x$	$x \cdot y = y \cdot x$
Associative Law	$x+(y+z) = (x+y)+z$	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$
Distributive Law	$x \cdot (y+z) = x \cdot y+x \cdot z$	$x+y \cdot z = (x+y) \cdot (x+z)$
Demorgan's Law	$(x+y)' = x' \cdot y'$	$(x \cdot y)' = x' + y'$
Absorption Law	$x + (x \cdot y) = x$	$x \cdot (x + y) = x$

Duality Theorem

Relation	Dual Relation
$A + 0 = A$	$A \cdot 1 = A$
$A + 1 = 1$	$A \cdot 0 = 0$
$A + A = A$	$A \cdot A = A$
$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$

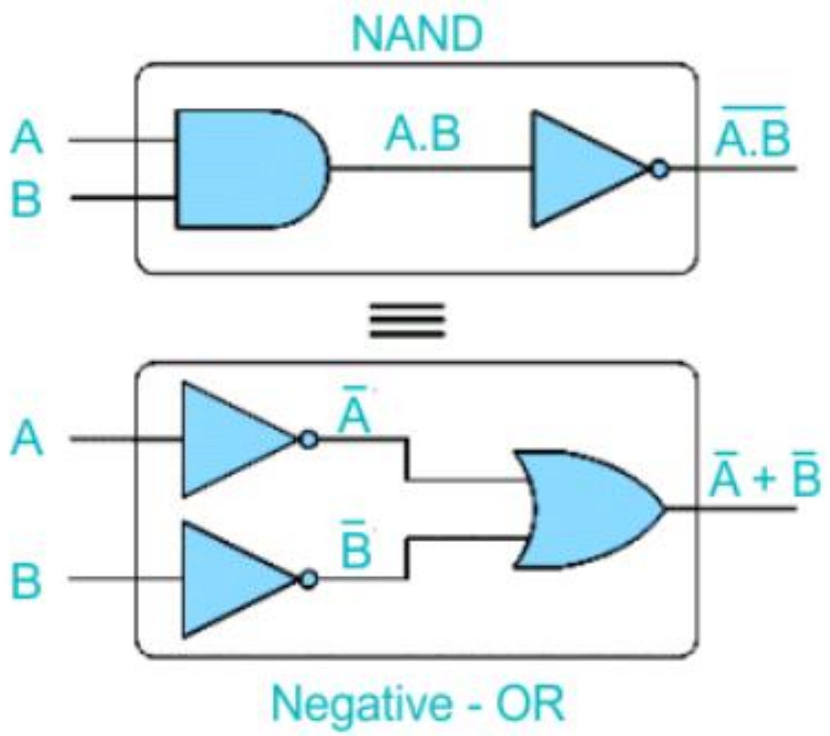
DE-MORGAN'S LAW

The complement of the product of all the terms is equal to the sum of the complement of each term. Likewise, the complement of the sum of all the terms is equal to the product of the complement of each term.

$$\overline{A\,B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A}\,\overline{B}$$

DEMORGAN'S LAW



A	B	$\overline{A \cdot B}$ $\stackrel{\checkmark}{=}$	$\bar{A}+\bar{B}$	$\overline{A+B}$ $\stackrel{\checkmark}{=}$	$\bar{A}\cdot\bar{B}$
0	0	1	1	1	1
0	1	1	1	0	0
1	0	1	1	0	0
1	1	0	0	0	0

ADVANTAGES OF USING BOOLEAN LAWS

- Use less number of gates
- Reduces Cost
- Reduces Power dissipation

Simplify:

Simplify: $A\overline{B}+\overline{(\overline{A}+\overline{B}+C.\overline{C})}$

Solution, $F= A\overline{B}+\overline{(\overline{A}+\overline{B}+C.\overline{C})}$

$= A\overline{B}+\overline{(\overline{A}+\overline{B}+0)}$

$= A\overline{B}+\overline{(\overline{A}+\overline{B})}$

$= A\overline{B}+(\overline{\overline{A}}.\overline{\overline{B}})$

$= A\overline{B}+A.B$

$= A(\overline{B}+B)$

$= A.1$

$= A$

$A.\overline{A}=0$

$A+0=A$

$\overline{A+B}=\overline{A}.\overline{B}$

$\overline{\overline{A}}=A$

$A+\overline{A}=1$

$A.1=A$

Simplification of Boolean Algebra

- $(A + B)(A + C) = A + BC$
- This rule can be proved as follows:
- $(A + B)(A + C) = AA + AC + AB + BC$ (Distributive law)

$= A + AC + AB + BC \quad (AA = A)$

$= A(1 + C) + AB + BC \quad (1 + C = 1)$

$= A.1 + AB + BC$

$= A(1 + B) + BC \quad (1 + B = 1)$

$= A.1 + BC \quad (A.1 = A)$

$= A + BC$

Simplify: $AB+\overline{A}B+A\overline{B}$

Solution, $F= AB+\overline{A}B+A\overline{B}$

$= B(A+\overline{A})+A\overline{B}$ — $A+\overline{A}=1$

$= B.1+A\overline{B}$ — $A.1=A$

$= B+A\overline{B}$ — $A+BC=(A+B)\ (A+C)$

$= (B+A)(B+\overline{B})$ — $A+\overline{A}=1$

$= (B+A).1$ — $A.1=A$

$= B+A$ — $A+B=B+A$

$= A+B$

Assessment 1

Identify the laws applied:

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1 + C + B) + BC \\
&= A \cdot 1 + BC \\
&= A + BC
\end{aligned}
$$


Thank you