DIGITAL ELECTRONICS:
PRODUCT OF SUM IN BOOLEAN ALGEBRA

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

PRODUCT OF SUM IN BOOLEAN ALGEBRA

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BOOLEAN EXPRESSION

- In Digital Electronics any logic circuit's output is the function of digital inputs.

- The relation between input and output can be represented using logic table or Boolean expressions.

This Boolean expression can be represented in two forms.

- Sum of Product (SOP)
- Product of Sum (POS)

The Product of Sum Form

- In sum of product form of expression, we perform **logical AND** operations on **different sum terms**.

$$Y = (A+B+C)\ (A+B+C')\ (A+B'+C')$$

The POS form can be in either:

(i) Non-canonical form.

(ii) Canonical form

Non-Canonical POS Form

In this form each sum term between may or may not contain all the variables of the function.

Examples:

$$F(A,B,C) = (A+B+C')(B'+C')(A'+C')$$

Canonical form

Canonical POS Form

In canonical POS form each Sum term contains all the variables of the function, where variables in each sum term can be in true form or complemented form.

Examples:

$$F(A,B,C) = (A+B+C')(A+B'+C')(A'+B+C')$$

The sum terms are called as **Maxterms**

POS

- With 'n' inputs namely X, Y, Z ,we get 2^n combinations.
- Here number of inputs are three and so 8 combinations

X	Y	Z	Minterms
0	0	0	X+Y+Z = M0
0	0	1	X+Y+Z' = M1
0	1	0	X+Y'+ Z = M2
0	1	1	X+Y'+Z' = M3
1	0	0	X'+Y+Z= M4
1	0	1	X'+Y+Z' = M5
1	1	0	X'+Y'+Z = M6
1	1	1	X'+Y'+Z' = M7

Canonical Product of Sum(POS)

Find the Canonical Product of Sum:

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Canonical Product of Sum

Find the Canonical Product of Sum:

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

For the table, the **canonical POS form** is:

$$F = \prod (M0, M4, M6, M7)$$

By expanding the above equation we can get the following function.

$$F = M0, M4, M6, M7$$

By substituting the maxterms in the above equation we can get the below expression

$$F = (X+Y+Z)\,(X'+Y+Z)(X'+Y'+Z)(X'+Y'+Z')$$

Non- Canonical Product of Sum

Find the Non- Canonical Product of Sum :

In the non-canonical Product of Sum, the sum terms are simplified.

$$F = (X+Y+Z) \ (X'+Y+Z)(X'+Y'+Z)(X'+Y'+Z')$$

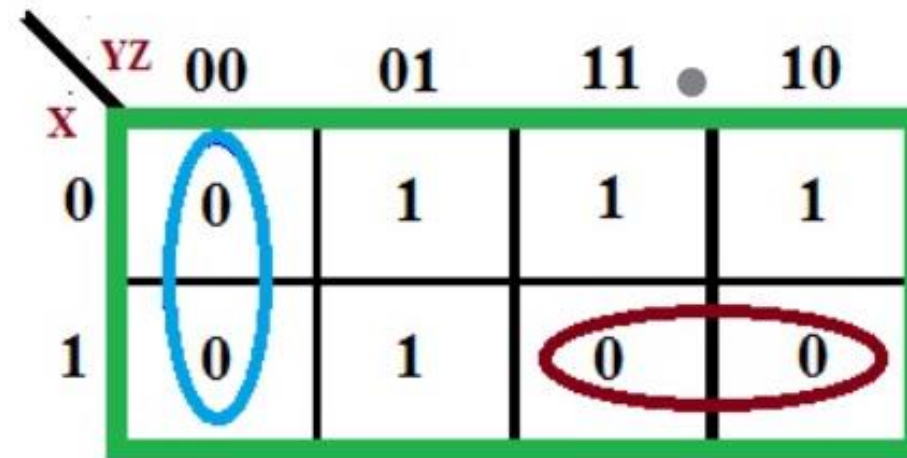$$F = (Y+Z) \ (X'+Y+Z) \ (X'+Y'+Z')$$

Consider the first two terms

$$= (X+Y+Z) \ (X'+Y+Z)$$

$$= XX'+XY+XZ+X'Y+YY+YZ+X'Z+YZ+ZZ$$

$$= 0+XY+XZ+X'Y+YY+YZ+X'Z+YZ+Z$$

$$= X \ (Y+Z) + X' \ (Y+Z) + Y(1+Z) + Z$$

$$= (Y+Z) \ (X+X') + Y \ (1) + Z$$

$$= (Y+Z) \ (0) + Y+Z$$

$$= Y+Z$$

Minimal Product of Sum

Find the Minimal Product of Sum :

- It is simply done by using **K-map (Karnaugh map)**.

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

F = (Y+Z) (X'+Y')

Schematic Design of Product of Sum

Canonical Form

F = (X+Y+Z) (X'+Y+Z)(X'+Y'+Z)(X'+Y'+Z')

Minimal Form
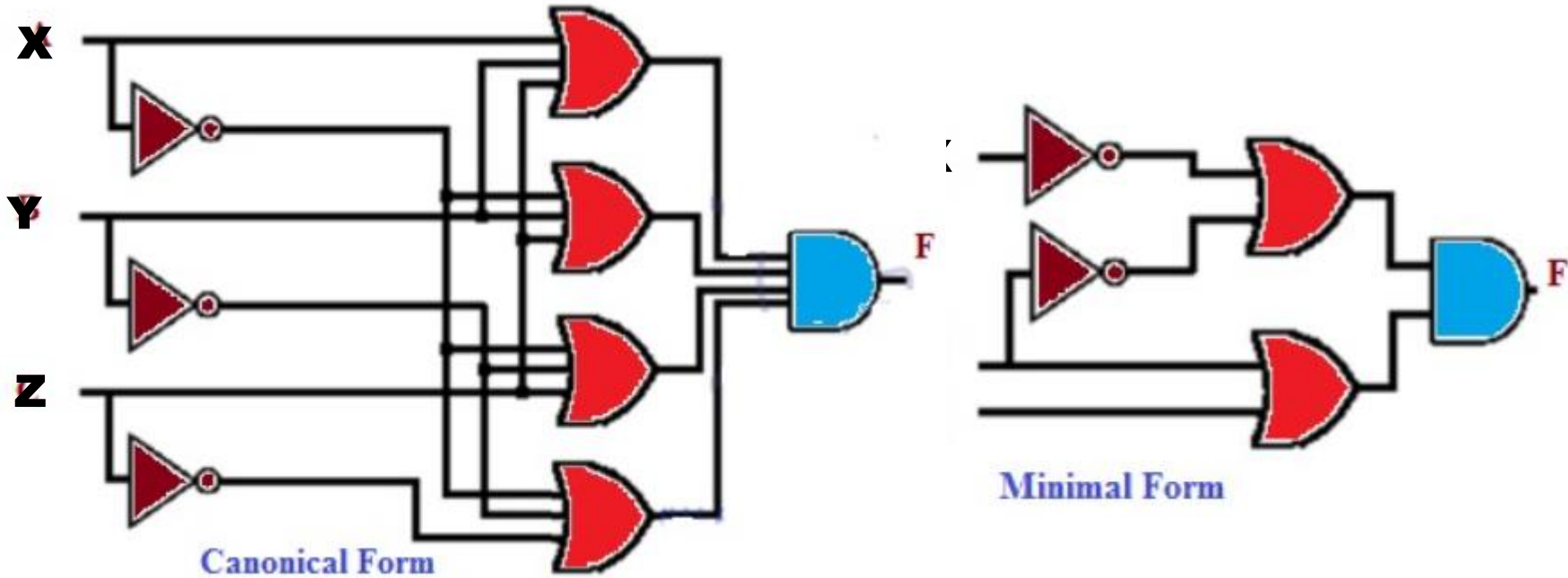
F = (Y+Z) (X'+Y')

Assessment

1. What is a maxterm?

   _____

2. Mention the difference between SOP and POS?

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