DIGITAL ELECTRONICS:
QUINE MCCLUSKEY METHOD

SNS COLLEGE OF ENGINEERING

Kurumbapalayam (PO), Coimbatore – 641 107

An Autonomous Institution

Accredited by NAAC – UGC with 'A' Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Quine-McCluskey Method

Dr.G.Arthy
Assistant Professor
Department of EEE
SNS College of Engineering

Quine-McCluskey Method

Methods for minimizing Boolean functions:

1. Boolean Laws
2. K-Map
3. Quine –McCluskey Method (or) Tabular Method

Quine-McCluskey Method

Quine-McCluskey Method is used to simplify Boolean expressions with 6 or more variables.

Used to design complex digital systems.

Quine-McCluskey Method

Key Terms

Implicants: Group of ones

Prime Implicants: Largest possible Group of Ones

Essential Prime Implicants: This is Prime Implicants having atleast one minterm that cannot be combined in any other way.

Procedure of Quine-McCluskey Tabular Method

Steps for simplifying Boolean functions using Quine-McCluskey tabular method.

Step 1 – Arrange the given min terms in an **ascending order** and make the groups based on the number of ones present in their binary representations. So, there will be **at most 'n+1' groups** if there are 'n' Boolean variables in a Boolean function or 'n' bits in the binary equivalent of min terms.

Step 2 – Compare the min terms present in **successive groups**. If there is a change in only one-bit position, then take the pair of those two min terms. Place this symbol '_' in the differed bit position and keep the remaining bits as it is.

Step 3 – Repeat step2 with newly formed terms till we get all **prime implicants**.

Procedure of Quine-McCluskey Tabular Method

- **Step 4** – Formulate the **prime implicant table**. It consists of set of rows and columns. Prime implicants can be placed in row wise and min terms can be placed in column wise. Place '1' in the cells corresponding to the min terms that are covered in each prime implicant.

- **Step 5** – Find the essential prime implicants by observing each column. If the min term is covered only by one prime implicant, then it is **essential prime implicant**. Those essential prime implicants will be part of the simplified Boolean function.

- **Step 6** – Reduce the prime implicant table by removing the row of each essential prime implicant and the columns corresponding to the min terms that are covered in that essential prime implicant. Repeat step 5 for Reduced prime implicant table. Stop this process when all min terms of given Boolean function are over.

Example

Simplify the following Boolean function, $f(W,X,Y,Z) = \sum m(2,6,8,9,10,11,14,15)$ using Quine-McCluskey tabular method.

Solution:

Step 1: (i) Write the Binary value for the corresponding decimal values
(ii) Make groups based on the number of ones present in their binary representations

Step 1: Write the Binary value for the corresponding decimal values $f(W,X,Y,Z) = \sum m(2,6,8,9,10,11,14,15)$

Decimal	Binary	No. of Ones
0	0000	0
1	0001	1
2	0010	1
3	0011	2
4	0100	1
5	0101	2
6	0110	2
7	0111	3

Decimal	Binary	No. of Ones
8	1000	1
9	1001	2
A(10)	1010	2
B(11)	1011	3
C(12)	1100	2
D(13)	1101	3
E(14)	1110	3
F(15)	1111	4

- The ascending order of these min terms based on the number of ones present in their binary equivalent is 2, 8, 6, 9, 10, 11, 14 and 15.

Step 1: Make groups based on the number of ones present in their binary representations

The given min terms are arranged into 4 groups based on the number of ones present in their binary equivalents.

Group Name	Min terms	W	X	Y	Z
GA1	2	0	0	1	0
	8	1	0	0	0
GA2	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
GA3	11	1	0	1	1
	14	1	1	1	0
GA4	15	1	1	1	1

Step 2:

The min terms, which are differed in only one-bit position from adjacent groups are merged. That differed bit is represented with this symbol, '-'. In this case, there are three groups and each group contains combinations of two min terms. The following table shows the possible **merging of min term pairs** from adjacent groups.

Group Name	Min terms	W	X	Y	Z
GB1	2,6	0	-	1	0
	2,10	-	0	1	0
	8,9	1	0	0	-
	8,10	1	0	-	0
GB2	6,14	-	1	1	0
	9,11	1	0	-	1
	10,11	1	0	1	-
	10,14	1	-	1	0
GB3	11,15	1	-	1	1
	14,15	1	1	1	-

Step 2: The successive groups of min term pairs, which are differed in only one-bit position are merged. That differed bit is represented with this symbol, '-'.

Group Name	Min terms	W	X	Y	Z
GB1	2,6,10,14	-	-	1	0
	2,10,6,14	-	-	1	0
	8,9,10,11	1	0	-	-
	8,10,9,11	1	0	-	-
GB2	10,11,14,15	1	-	1	-
	10,14,11,15	1	-	1	-

Step 3:

In the above table, there are two groups and each group contains combinations of four min terms. Here, these combinations of 4 min terms are available in two rows. So, we can remove the repeated rows. The reduced table after removing the redundant rows is shown below.

Group Name	Min terms	W	X	Y	Z
GC1	2,6,10,14	-	-	1	0
	8,9,10,11	1	0	-	-
GC2	10,11,14,15	1	-	1	-

The **prime implicants** are YZ', WX' & WY.

Step 4:

The **prime implicant table** is shown below.

Group Name	Min terms	W	X	Y	Z
GC1	2,6,10,14	-	-	1	0
	8,9,10,11	1	0	-	-
GC2	10,11,14,15	1	-	1	-

The **prime implicants** are YZ', WX' & WY.

Min terms / Prime Implicants	2	6	8	9	10	11	14	15
YZ'	1	1			1		1	
WX'			1	1	1	1		
WY					1	1	1	1

Step 5: To find Essential Prime Implicants

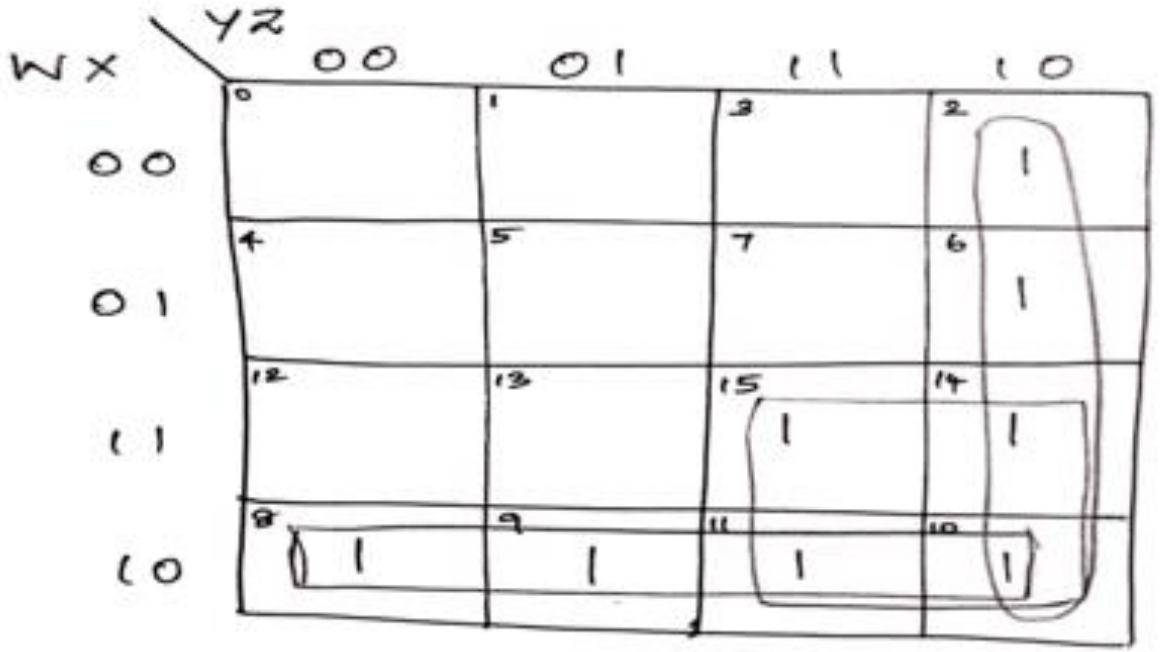
Min terms / Prime Implicants	2	6	8	9	10	11	14	15
YZ′	1	1			1		1	
WX′			1	1	1	1		
WY					1	1	1	1

We got three prime implicants and all the three are essential. Therefore, the **simplified Boolean function** is

$$f(W, X, Y, Z) = YZ' + WX' + WY.$$

Verification Using K-Map

$f(W,X,Y,Z) = \sum m(2,6,8,9,10,11,14,15)$



Simplified form :

$f(W,X,Y,Z) = YZ' + WX' + WY$

Assessment

1. What is the advantage of Quine McCluskey Method?

2. List the techniques available to Simplify Boolean Expression.


Thank
you