PROBABILITY THEORY AND RANDOM PROCESS
Unit-I
PART A

1. Write down the axioms of Probability
2. State Baye's theorem.
3. Define discrete and Continuous random variable.
4. Test Whether $\begin{cases} |x|, -1 < x < 1 \\ 0, Otherwise \end{cases}$ can be the probability density function of continuous random variable?
5. A CRV X that can assume any value between x=2 and x=5 has a density function given by f(x) = k (1+x). Find k.
6. Find the value of 'K' for a continuous random variable X whose probability density function is given by $f(x) = Kx^2e^{-x}; x \geq 0.$
7. X and Y are independent random variables with variance 2 and 3. Find the variance of 3X+4Y.
8. The mean of a Binomial distribution is20 and S.D is 4. Determine the parameters of the distribution.
9. Determine the Binomial distribution for which the mean is 4 and variance is 3.
10. Comment on the following "The mean of a Binomial distribution is 3 & Variance is 4".
11. Write the mean and variance of Binomial distribution.
12. Define Poisson distribution and write its mean and variance
13. Write the mean and variance of Exponential distribution.
14. State Memoryless property of Exponential Distribution
15. Write any two properties of normal distribution.

PART B

1. A random variable x has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

(i) Find the value of K
(ii) Evaluate P[X<6] and P[X≥ **6**)
(iii) If P[X≤ $\boldsymbol{C}$)>1/2 find minimum value of C
(iv) Evaluate $P[1.5 < x < 4.5/x > 2]$

2. A random variable X has the following probability distribution.

x	-2	1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

a. Find K
b. Evaluate P(x < 2) and P(-2 < x □ 2)
c. Find the Cumulative distribution of x.
d. Evaluate the mean of x.

3. The probability mass function of a discrete R. V X is given in the followingtable

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) the value of a , (ii) $P(X < 3)$, (iii) Mean of X, (iv) Variance ofX.

4. A Random variable X has the following probability function:

x	0	1	2	3	4
P(x)	K	3K	5K	7K	9K

Find K, P[X≥ 3] $and\ P(0 < X < 4)$.

5. The Probability function of an infinite discrete distribution is given by $P[X = j] = \frac{1}{2^j}$, j= 1,2,……∞. Find the Mean and variance of the distribution. Also find P[X is Even] and P[X≥ 5].
6. If the density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax, 0 \le x \le 1 \\ a, 1 \le x \le 2 \\ 3a - x, 2 \le x \le 3 \\ 0, elsewhere \end{cases}$$

   (i) Find the value of 'a'
   (ii) Find the CDF of x
   (iii) Compute P(x≤1.5) and p(x>1.5)
7. A continuous random variable X that can assume any value between X=2 and X=5 has the density function given by f(x)=k(1+x). Find P[X<4], P[3<X<4].
8. A continuous random variable X having the probability density function $f(x) = 3x^2, 0 \le x \le 1$. Find K, such that P[X>K]=0.5.
9. A continuous random variable X has the distribution function F[x]=$\begin{cases} 0, x < 1 \\ k(x-1)^4, 1 \le x \le 3 \\ 1, x > 3 \end{cases}$

   Find K, Probability density function of f(x) , P[x<2].
10. In a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \dfrac{x^2}{3} \quad , \quad -1 < x < 2 \\ 0, Otherwise \end{cases}$$

11. The p.d.f of a continuous random variable X is f(x)=K$e^{-|x|}$. $Find\ K\ and\ F[x]$.
12. Find the MGF of Binomial distribution. Hence find its Mean and variance.
13. Find the MGF of Poisson distribution and hence find its mean and variance
14. Find the MGF of Geometric distribution and hence find its mean and variance.
15. Find the MGF of Uniform distribution and hence find its mean and variance.
16. Find the MGF of Exponential distribution and hence find its mean and Variance. Also provethe memory less property of the exponential distribution.
17. Find the MGF of Normal distribution & hence find its mean and variance

18. A bolt is manufactured by 3 machines A, B, and C. A turns out twice as many items as B andmachines B and C produce equal number of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and1 is chosen from this pile. What is the probability that it is defective?
19. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ballis taken at random from the latter. What is the probability that it is a white ball?
20. Out of 800 families with 4 children each how many families would be expected to have
    i. 2 Boys and 2 Girls
    ii. At least 1 boy
    iii. At most 2 girls
    iv. Children of both genders,

    Assume equal probabilities for boys and girls.
21. A machine manufacturing screws is known to produce 5% defective, In a random sample of 15 screws, what is the probability that they are (i) exactly 3 defectives, (ii) not more than 3 defectives
22. A Manufacture of cotton pins that 5% of his product is defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?
23. If X is Poisson variance such that $P(x = 2) = 9P(X = 4) + 90P(X = 6)$, find the mean and variance.
24. The number of monthly breakdowns of a computer is a random variable, having a Poisson distribution with mean equal to 1.8. find the probability that this computer will function for amonth.
    i. Without a breakdown
    ii. With only one breakdown
    iii. With at least one breakdown
25. A die is Cast until 6 appears what is the probability that it must cast more than 5 times?
26. If the probability that an applicant for a drivers license will pass the road test on any given train 0.8. what is the probability that he will finally pass the test
    i) On 4^{th} trail
    ii) In fewer than 4 trails.
27. X is uniformly distributed random variable with Mean 1 and Variance 4/3, find P(X<0).
28. A random variable X has a uniform distribution over (-3,3) Compute
    i) P(x<2)
    ii) P(|x|<2)
    iii) P(|x-2|<2)
    iv) Find K for which P(X>K)=1/3.
29. Buses arrive at a specified bus at 15min intervals. Starting at 7am , 7.15am, 7.30am…… if a passenger arrive at a bus stop at a random time which is uniformly distributed between 7 and 7.30am, find the probability that he waits
    i) Less than 5 minutes
    ii) At least 12 minutes for a bus.

30. The length of time a person speaks over phone follows a exponential distribution with mean 6. What is the probability that the person will talk for
    i) more than 8 minutes
    ii) between 4 and 8 minutes
31. The mileage which car owners get with a certain kind of radial tire is the random variable having a exponential distribution with mean 40,000km. Find the probability that one of these tire will last
    i) At least 20,000km
    ii) At most 30,000km
32. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda=1/2$
    (i)What is the probability that the repairs time exceeds 2 hour?
    (ii) What is the conditional probability that the repair takes 10 hour given thatits duration exceeds 9 hour?

Unit-II

PART A

1. The joint probability mass function of a two dimensional random variable (X,Y)is given by p(x,y)= k(2x+y), x =1,2 y=1,2, where K is constant. Find the value of k.
2. Let X and Y have the joint p.m.f

Y/X	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

   Find P(X+Y>1).
3. Given the joint probability function of X and Y has f(x)= $\begin{cases} \frac{1}{6}, & 0 < x < 2, 0 < y < 3 \\ 0, & Otherwise \end{cases}$ find the Marginal density function of X.
4. The joint pdf of a random variable (X,Y) is $f(x,y) = ke^{-(2x+3y)}; x > 0, y > 0$. Find the value of k.
5. The joint pdf of a random variable (X,Y) is $f(x,y) = kxe^{-y}; 0 < x < 2, y > 0$. Find the value of k.
6. The joint pdf of random variable (X,Y) is given as $f(x,y) = \frac{1}{x}, 0 < x < y < 1$ Find the marginal pdf of Y.
7. If f(x,y) = $\begin{cases} 8xy, 0 < x < 1, \ 0 < y < x \\ 0, \quad otherwise \end{cases}$ is the joint probability density function of X and Y, find f(y/x).

PART B

1. The joint probability mass function of (X Y), is given by p(x,y)=k(2x+3y) x = 0,1,2; y=1,2,3. Find k and all the marginal and conditional probability distributions. Also find the probability distribution of X+Y

2. The joint probability mass function of (X Y), is given by $f(x, y) = k(x + 2y)$
   x = 0,1,2; y=0,1,2. Find all the marginal and conditional probability distributions
3. The joint probability mass function of (X Y), is given by $f(x, y) = \frac{x+2y}{27}$
   x = 0,1,2; y=0,2,3. Find all the marginal and conditional probability distributions.
4. The joint probability mass function of (X Y), is given by p(x,y)= $\frac{1}{72}$ (2x+3y)
   x = 0,1,2; y=1,2,3. Find all the marginal and conditional probability distributions.

5. The joint pdf of the random variable (X,Y) is given by $f(x, y) = Kxye^{-(x^2+y^2)}, x. > 0, y > 0.$ Find the value of K and also prove that X and Y are independent.
6. If the joint pdf of a two-dimensional random variable (X, Y) is given by
   $$f(x, y) = \begin{cases} 8xy, 0 < x < y: 0 < y < 1 \\ 0 \ , \ Otherwise \end{cases}$$
   Find $(i)\ Are\ X\ and\ Y\ independent$?
7. If the joint pdf of a two-dimensional random variable (X, Y) is given by
   $$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, 0 < x < 1: 0 < y < 2 \\ 0 \ , \ Otherwise \end{cases}$$ Find
   $(i)\ Are\ X\ and\ Y\ independent$?
   $(ii) Find\ the\ condition\ probability\ density\ function\ of\ X\ given\ Y$
   $(iii) p\left(y < \frac{1}{2} / x < \frac{1}{2}\right)$

8. If $f(x, y) = \begin{cases} k(1 - x^2y) , 0 \le x, y \le 1 \\ 0 \ , \ Otherwise \end{cases}$.
   i) Find K
   ii) Obtain Marginal p.d.f of X and Y
9. Given the joint pdf of X and Y $f(x, y) = \begin{cases} cx(x - y), 0 < x < 2, -x < y < x \\ 0 otherwise \end{cases}$,
   i. Evaluate c
   ii. Find Marginal pdf of X and Y.
   iii. Find the conditional density of Y/X.