PSG 8.39

✓ Root angle : $\delta_{f1} = \delta_1 - \theta_{f1} = 26.57° - 3.07° = 23.5°$; and

$\delta_{f2} = \delta_2 - \theta_{f2} = 63.43° - 3.07° = 60.36°$

✓ Virtual number of teeth : $z_{v1} = 23$; and $z_{v2} = 90$.

(4)

Example 7.12 *Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 r.p.m. and the speed of the gear wheel shaft is 120 r.p.m. Pinion is of steel and wheel of cast iron. Each gear is expected to work 2 hours / day for 10 years. The drive transmits 9.37 kW.*

10×365
3650×2
7300

Given Data : $\theta = 90°$; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; $P = 9.37$ kW.

To find : Design the bevel gear drive.

Solution : Since the materials of pinion and gear are different, we have to design the pinion first and check the gear.

1. ***Gear ratio :*** $i = \frac{N_1}{N_2} = \frac{360}{120} = 3$

Pitch angles : $\tan \delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

Tensile stress Yield point stress

2. ***Material selection :*** Pinion – C45 Steel, $\sigma_u = 700$ N/mm² and $\sigma_y = 360$ N/m²

1.40 Gear – CI grade 35, $\sigma_u = 350$ N/mm², from Table 5.3.

3. ***Gear life*** in hours = (2 hours/day) × (365 days / year × 10 years) = 7300 hours

∴ Gear life in cycles, $N = 7300 \times 360 \times 60 = 15.768 \times 10^7$ cycles

4. ***Calculation of initial design torque [M_t] :***

We know that, 8.15 $[M_t] = M_t \times K \times K_d$

where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 9.37 \times 10^3}{2\pi \times 360} = 248.6$ N-m, and

$K \cdot K_d = 1.3$, initially assumed.

∴ $[M_t] = 248.6 \times 1.3 = 323.28$ N-m

5. ***Calculation of E_{eq}, [σ_b] and [σ_c] :***

8.14 ✓ *To find E_{eq} :* $E_{eq} = 1.7 \times 10^5$ N/mm², from Table 5.20.

✓ *To find [σ_{b1}] :* We know that the design bending stress for pinion,

$$[\sigma_{b1}] = \frac{1.4\, K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1},$$ for rotation in one direction

where 8.19 $K_{bl} = 1$, for HB ≤ 350 and N ≥ 10^7, from Table 5.14,

8.19 $K_\sigma = 1.5$, for steel pinion, from Table 5.15,

8.19 $n = 2.5$, steel hardened, from Table 5.17.

8.19 $\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$, for forged steel, from Table 5.16.

$= 0.25(700 + 360) + 50 = 315$ N/mm²

∴ $[\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6$ N/mm²

✓ *To find [σ_{c1}] :* We know that the design contact stress for pinion,

8.16 $[\sigma_{c1}] = C_R \cdot HRC \times K_{cl}$

where 8.16 $C_R = 23$, from Table 5.18,

8.16 $HRC = 40$ to 55, from Table 5.18, and

8.17 $K_{cl} = 1$, for steel pinion, HB ≤ 350 and N ≥ 10^7, from Table 5.19.

∴ $[\sigma_{c1}] = 23 \times 50 \times 1 = 1150$ N/mm²

6. ***Calculation of cone distance (R) :***

1st formula

We know that, $R \geq \psi_y \sqrt{i^2+1}\ \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$ PSG 8.13

where 8.15 $\psi_y = R/b = 3$, initially assumed.

∴ $R \geq 3\sqrt{3^2+1}\ \sqrt[3]{\left[\frac{0.72}{(3-0.5)\,1150}\right]^2 \times \frac{1.7 \times 10^5 \times 323.28 \times 10^3}{3}}$

≥ 99.36

or $R = 100$ mm.

7. Assume $z_1 = 20$; Then $z_2 = i \times z_1 = 3 \times 20 = 60$

Virtual number of teeth : $z_{v1} = \frac{z_1}{\cos \delta_1} = \frac{20}{\cos 18.44°} \approx 22$; and

$z_{v2} = \frac{z_2}{\cos \delta_2} = \frac{60}{\cos 71.56°} \approx 190.$

8. ***Calculation of transverse module (m_t) :***

We know that,

$$m_t = \frac{R}{0.5\sqrt{z_1^2 + z_2^2}} = \frac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162 \text{ mm}$$

BEVEL

From Table 5.8, the nearest higher standard transverse module is 4 mm.

9. *Revision of cone distance (R) :*

We know that, $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 4 \sqrt{20^2 + 60^2} = 126.49$ mm

10. *Calculation of b, m_{av}, d_{1av}, v and ψ_y :*

✓ Face width (b) : $b = \frac{R}{\psi_y} = \frac{126.49}{3} = 42.16$ mm

8.15

✓ Average module (m_{av}) : $m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 4 - \frac{42.16 \times \sin 18.44°}{20}$

$= 3.333$

✓ Average pcd of pinion (d_{1av}) : $d_{1av} = m_{av} \times z_1 = 3.333 \times 20 = 66.66$ mm

✓ Pitch line velocity (v) : $v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256$ m/s

✓ $\psi_y = \frac{b}{d_{1av}} = \frac{42.16}{66.66} = 0.632$

11. *IS quality 6 bevel gear* is assumed, from Table 5.22.

12. *Revision of design torque [M_t] :*

We know that, $[M_t] = M_t \times K \times K_d$

where 8.15 $K = 1.1$, from Table 7.2, and

8.16 $K_d = 1.35$, from Table 5.12.

$\therefore$ $[M_t] = 248.6 \times 1.1 \times 1.35 = 369.28$ N-m

13. *Check for bending of pinion :* We know that the induced bending stress,

8.13A $\sigma_{b1} = \frac{R\sqrt{i^2+1}\ [M_t]}{(R-0.5\, b)^2 \times b \times m_t \times y_{v1}}$ 4th formula

where 8.18 $y_{v1} = 0.402$, for $z_{v1} = 22$, from Table 5.13

$\therefore$ $\sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49 - 0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402} = 196.09$ N/mm²

We find $\sigma_{b1} > [\sigma_{b1}]$. Thus the design is unsatisfactory.

Trial 2 : Now we will try with increased transverse module 5 mm. Repeating from Step 1 again, we get

$R = 0.5 \times m_t \times \sqrt{z_1^2 + z_2^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11$ mm

$b = \frac{R}{\psi_y} = \frac{158.11}{3} = 52.7$ mm

$m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 5 - \frac{52.7 \times \sin 18.44}{20} = 4.166$ mm

$d_{1av} = m_{av} \times z_1 = 4.166 \times 20 = 83.33$ mm

8.15 $v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = 1.57$ m/s

$\psi_y = \frac{b}{d_{1av}} = \frac{52.7}{83.33} = 0.632$

IS quality 6 bevel gear is assumed.

$K = 1.1$; $K_d = 1.35$.

$M_t = 248.6 \times 1.1 \times 1.35 = 369.28$ N-m

$\therefore$ $\sigma_{b1} = \frac{158.11\sqrt{3^2+1} \times 369.28 \times 10^3}{(158.11 - 0.5 \times 52.7)^2 \times 52.7 \times 5 \times 0.402} = 100.4$ N/mm²

Now we find $\sigma_{b1} < [\sigma_{b1}]$, thus ***the design is satisfactory.***

14. *Check for wearing of pinion :* We know that the induced contact stress,

$\sigma_{c1} = \left(\frac{0.72}{R - 0.5\, b}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i\, b} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}}$ 8.13 2nd form. PSG

$= \left[\frac{0.72}{158.11 - 0.5 \times 52.7}\right]\left[\frac{\sqrt{(3^2+1)^3}}{3 \times 52.7} \times 1.7 \times 10^5 \times 369.28 \times 10^3\right]^{\frac{1}{2}}$

$= 612.33$ N/mm²

We find $\sigma_{c1} < [\sigma_{c1}]$. Thus ***the design is satisfactory for pinion.***

15. *Check for gear (i.e., wheel) : Gear material :* CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel, $N = \frac{N_{pinion}}{3} = \frac{15.768 \times 10^7}{3} = 5.256 \times 10^7$ cycles

✓ ***To find [σ_{b2}] :*** We know that the design bending stress for gear,

$[\sigma_{b2}] = \frac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1}$

where

8.20 $K_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{5.256 \times 10^7}} = 0.832$, from Table 5.14,

8.19 $K_\sigma = 1.2$, from Table 5.15,

8.19 $n = 2$, from Table 5.17.

$\sigma_{-1} = 0.45\, \sigma_u = 0.45 \times 350 = 157.5$ N/mm²

BEVEL

$$\therefore \quad [\sigma_{b2}] = \frac{1.4 \times 0.832}{2 \times 1.2} \times 157.5 = 76.44 \text{ N/mm}^2$$

✓ *To find* $[\sigma_{c2}]$: We know that the design contact stress for gear,

$$[\sigma_{c2}] = C_B \times HB \times K_{cl}$$

where 8.16 C_B = 2.3, from Table 5.18,

8.16 HB = 200 to 260, from Table 5.18, and

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256 \times 10^7}} = 0.758$$

$$\therefore \quad [\sigma_{c2}] = 2.3 \times 260 \times 0.758 = 453.284 \text{ N/mm}^2$$

(a) Check for bending of gear : The induced bending stress for gear can be calculated using the relation

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

where 8.18 y_{v1} = 0.402, for z_{v1} = 22, from Table 5.13, and

8.18 $y_{v2} \approx$ 0.520, for z_{v2} = 190, from Table 5.13.

$$\therefore \quad 100.4 \times 0.402 = \sigma_{b2} \times 0.520$$

or σ_{b2} = 77.6 N/mm²

We find σ_{b2} is almost equal to $[\sigma_{b2}]$. Thus the design is okay and it can be accepted.

(b) Check for wearing of gear : Since the contact area is same,

$$\sigma_{c2} = \sigma_{c1} = 612.33 \text{ N/mm}^2$$

We find $\sigma_{c2} > [\sigma_{c2}]$. It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 360 BHN. Then we get

$$[\sigma_{b2}] = 2.3 \times 360 \times 0.758 = 627.62 \text{ N/mm}^2.$$

Now we find $\sigma_{b2} < [\sigma_{b2}]$, thus the *design is safe and satisfactory.*

BEVEL Stop

REVIEW AND SUMMARY

- ✓ *Bevel gears are used to transmit power between two intersecting shafts.*
- ✓ *Types : 1. Straight bevel gears ; 2. Spiral bevel gears ; 3. Zerol bevel gears ; and 4. Hypoid gears.*
- ✓ ***Classification based on pitch angle :*** *1. Crown gear; 2. Internal bevel gear; and 3. Mitre gears.*
- ✓ *The bevel gear nomenclature and its kinematics are presented in the beginning of the chapter.*

- ✓ ***Virtual number of teeth :*** $z_v = \dfrac{z}{\cos \delta}$
- ✓ *Tooth proportions and basic dimensions of bevel gears are tabulated in Table 7.1.*
- ✓ ***Force analysis on bevel gears :***

Three components of the resultant force on the gear tooth are :

1. *Tangential component :* $F_t = \dfrac{2\,M_t}{d_{1m}} = \dfrac{M_t}{r_m}$

   where r_m = *Mean radius of the pinion* $= \dfrac{d_1}{2} - \dfrac{b \cdot \sin \delta_1}{2}$

2. *Radial component :* $F_r = F_t \times \tan \alpha \times \cos \delta$
3. *Axial component :* $F_a = F_t \times \tan \alpha \times \sin \delta$

- ✓ ***Two methods of designing a bevel gear :*** *1. Bevel gear design using Lewis and Buckingham's equations; and 2. Bevel gear design based on gear life.*
- ✓ *The step by step procedure for the above said two methods are presented with sufficient illustrative problems.*
- ✓ ***Lewis beam strength for bevel gears :***

Beam strength, $F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\dfrac{R-b}{R}\right)$

where R = *Cone distance* $= 0.5\, m_t \sqrt{z_1^2 + z_2^2}$

- ✓ ***Buckingham's equation for bevel gears :***

(i) *Dynamic load,* $F_d = F_t + \dfrac{21\,v\,(bc + F_t)}{21\,v + \sqrt{bc + F_t}}$

(ii) *Wear tooth load,* $F_w = \dfrac{0.75\, d_1 \times b \times Q' \times K_w}{\cos \delta_1}$

REVIEW QUESTIONS

1. Under what situation, bevel gears are used ?
2. How are the bevel gears classified ? Explain with sketches.
3. What do you mean by spiral bevel gear ? What is its advantage over straight bevel gear ?
4. What is a zerol bevel gear ?
5. What is a hypoid gear ? Why is it used in automobiles ?
6. Differentiate an angular gear and a mitre gear.
7. With neat sketch, explain the nomenclature of a straight bevel gear.