ELECTROMAGNETIC FIELDS AND WAVES

FIELD DUE TO A SHEET OF CHARGE

A thin, infinite sheet uniformly charged with surface charge density σ (C/m²)

Consider an infinite charged sheet in the xy-plane.

Surface charge density: σ (C/m²)

A small element dQ on the sheet creates a small electric field dE.

$$dQ = \rho_S \, dS = \rho_S \, r \, dr \, d\phi$$

FIELD DUE TO A SHEET OF CHARGE

$$d\bar{\mathbf{E}} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{\mathbf{a}}_R$$

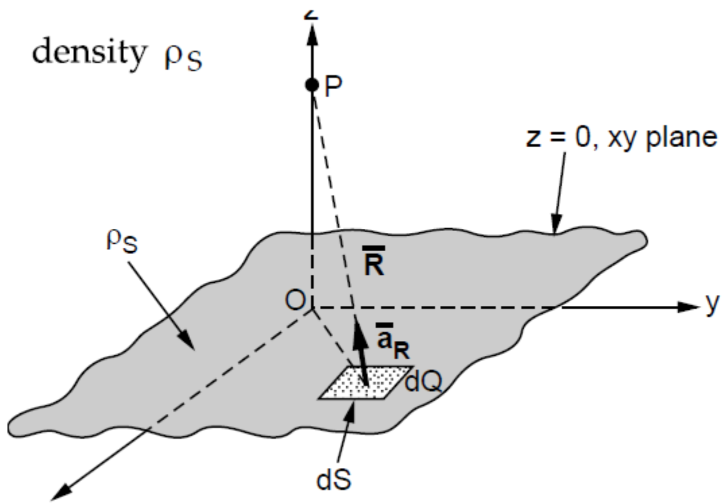
$$= \frac{\rho_S\, r\, dr\, d\phi}{4\pi\varepsilon_0 R^2}\,\bar{\mathbf{a}}_R$$

$$\bar{\mathbf{E}} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty} d\bar{\mathbf{E}} = \int_{0}^{2\pi}\int_{0}^{\infty} \frac{\rho_S\, r\, dr\, d\phi}{4\pi\varepsilon_0 \left(r^2+z^2\right)^{3/2}}\,(z\,\bar{\mathbf{a}}_z)$$

$$= \int_0^{2\pi} \frac{\rho_S}{4\pi\varepsilon_0} d\phi \ z \ \bar{\mathbf{a}}_z \left[-\frac{1}{u} \right]_z^{\infty} \qquad \ldots \text{as} \int \frac{1}{u^2} = \int u^{-2} = \frac{u^{-1}}{-1} = -\frac{1}{u}$$

$$= \frac{\rho_S}{4\pi\varepsilon_0} [\phi]_0^{2\pi} (z\,\bar{\mathbf{a}}_z) \left[-\frac{1}{\infty} - \left(-\frac{1}{z} \right) \right] = \frac{\rho_S}{4\pi\varepsilon_0} (2\pi)\ \bar{\mathbf{a}}_z$$

$$\bar{\mathbf{E}} = \frac{\rho_S}{2\varepsilon_0} \bar{\mathbf{a}}_z \ \ \text{V/m} \qquad \ldots \text{For points above xy plane}$$




Thank you