ELECTROMAGNETIC FIELDS AND WAVES

Numerical Problems

Verify that the vectors A = $4\,\bar{a}_x - 2\,\bar{a}_y + 2\,\bar{a}_z$ and

$\bar{B} = -6\,\bar{a}_x + 3\,\bar{a}_y - 3\,\bar{a}_z$ are parallel to each other.

The two vectors are parallel if their cross product is zero.

$$\bar{A} = 4\,\bar{a}_x - 2\,\bar{a}_y + 2\,\bar{a}_z \qquad \bar{B} = -6\,\bar{a}_x + 3\,\bar{a}_y - 3\,\bar{a}_z$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 4 & -2 & 2 \\ -6 & 3 & -3 \end{vmatrix} = \bar{a}_x\,[6 - 6] - \bar{a}_y\,[-12 + 12] + \bar{a}_z\,[12 - 12] = 0$$

The vectors are parallel to each other.

Two vectorial quantities $A = 4\,i + 3\,j + 5\,k$ and $B = i$ □$2\,j + 2\,k$ are known to be oriented in two unique directions. Determine the angular separation between them.

$$\theta = \cos^{-1}\left[\frac{\overline{A}\bullet\overline{B}}{\left|\overline{A}\right|\left|\overline{B}\right|}\right] = \cos^{-1}\left[\frac{(4\times 1)+(3\times -2)+(5\times 2)}{\sqrt{50}\times\sqrt{9}}\right] = \mathbf{67.844}^{\circ}$$

In XY plane, Q_1 = 100 μC at (2, 3)m, experiences a repulsive force of 7.5 N because of Q_2 at (10, 6) m. Find Q_2.

The force is given by,

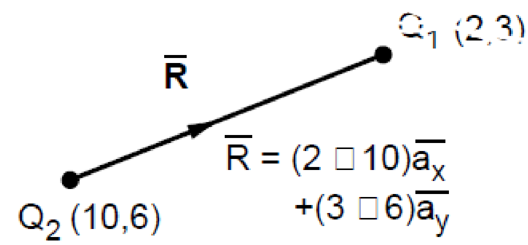

Fig. 1.2

$$\bar{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\,\bar{a}_R$$

$$\bar{R} = -8\,\bar{a}_x - 3\,\bar{a}_y, \quad |\bar{R}| = \sqrt{73}$$

$$\bar{F} = \frac{100\times10^{-6}\times Q_2\times[-8\,\bar{a}_x - 3\bar{a}_y]}{4\pi\times 8.854\times10^{-12}\times73\times\sqrt{73}} \qquad \ldots\ \bar{a}_R = \frac{\bar{R}}{|\bar{R}|}$$

$$\bar{F} = 1441\ Q_2\,[-8\,\bar{a}_x - 3\bar{a}_y]$$

$$|\bar{F}| = 1441\ Q_2\,\sqrt{8^2+3^2} = 7.5\ \text{N} \quad \text{given}$$

$$Q_2 = \frac{7.5}{1441\times\sqrt{73}} = \mathbf{609.166\ \mu C} \qquad \ldots\text{Positive}$$

A vector field is given by the expression $\overline{F} = \frac{1}{r}\overline{a}_r$ *in the spherical co-ordinates. Determine* $\overline{F}$ *in cartesian form at a point x = 1, y = 1 and z = 1 unit.*

Solution : $\overline{F} = \frac{1}{r}\,\bar{a}_r$ ... Spherical co-ordinates

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} F_r \\ F_\theta \\ F_\phi \end{bmatrix}$$

But $F_\theta = F_\phi = 0$ and $F_r = \frac{1}{r}$

$\therefore \quad F_x = \sin\theta\cos\phi\, F_r = \frac{1}{r}\sin\theta\cos\phi$

$\therefore \quad F_y = \sin\theta\sin\phi\, F_r = \frac{1}{r}\sin\theta\sin\phi$

$\therefore \quad F_z = \cos\theta\, F_r = \frac{1}{r}\cos\theta$

At x = 1, y = 1 and z = 1

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{3}, \quad \theta = \cos^{-1}\left[\frac{z}{r}\right] = 54.735^\circ, \quad \phi = \tan^{-1}\frac{y}{x} = 45^\circ$$

$\therefore \quad F_x = 0.333, \; F_y = 0.333, \; F_z = 0.4082$

$\therefore \quad \overline{F} = 0.333\,\bar{a}_x + 0.333\,\bar{a}_y + 0.4082\,\bar{a}_z$


Thank you