ELECTROMAGNETIC FIELDS AND WAVES

Field Due to Continuous Volume Charge Distribution

In electrostatics, when charge is continuously distributed over a **volume**, the electric field at a point is found by integrating over the entire charge distribution.

Instead of discrete charges, we consider **volume charge density** (ρ_v)

Volume Charge Density (ρ_v)

The charge per unit volume at a given point in space.

$$\rho_v = \frac{dq}{dV}$$

where:

- ρ_v → Volume charge density (C/m³)
- dq → Small charge element (C)
- dV → Small volume element (m³)

Expression for Electric Field

From **Coulomb's Law**, the differential electric field due to an infinitesimal charge element dq is:

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

Since dq is distributed over a volume, we replace it with $\rho_v dV$:

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_v dV}{r^2}\hat{r}$$

To find the **total electric field** at a point, integrate over the entire charge distribution:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_v dV}{r^2}\hat{r}$$

V is the volume containing the charge distribution.

$\hat{r}$ is the unit vector from the charge element to the point of interest.

r is the distance between the charge element and the field point.

Electric Field Due to a Uniformly Charged Sphere

For a sphere of radius R with uniform charge density ρ_v, total charge is:

$$Q = \int_V \rho_v dV = \rho_v \frac{4}{3}\pi R^3$$

Using **Gauss's Law**, the electric field is:

- **Inside the sphere ($r < R$):**

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\rho_v 4\pi r^3/3}{r^2}$$

$$E = \frac{\rho_v}{3\varepsilon_0} r$$

- **Outside the sphere ($r > R$):**

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Electric Field Due to an Infinitely Long Uniformly Charged Cylinder

Using **Gauss's Law**, for a cylinder of radius R with uniform charge density ρ_v:

- **Inside the cylinder** ($r < R$):

$$E = \frac{\rho_v r}{2\varepsilon_0}$$

- **Outside the cylinder** ($r > R$):

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where $\lambda = \rho_v \pi R^2$ is the **linear charge density**.


Thank you