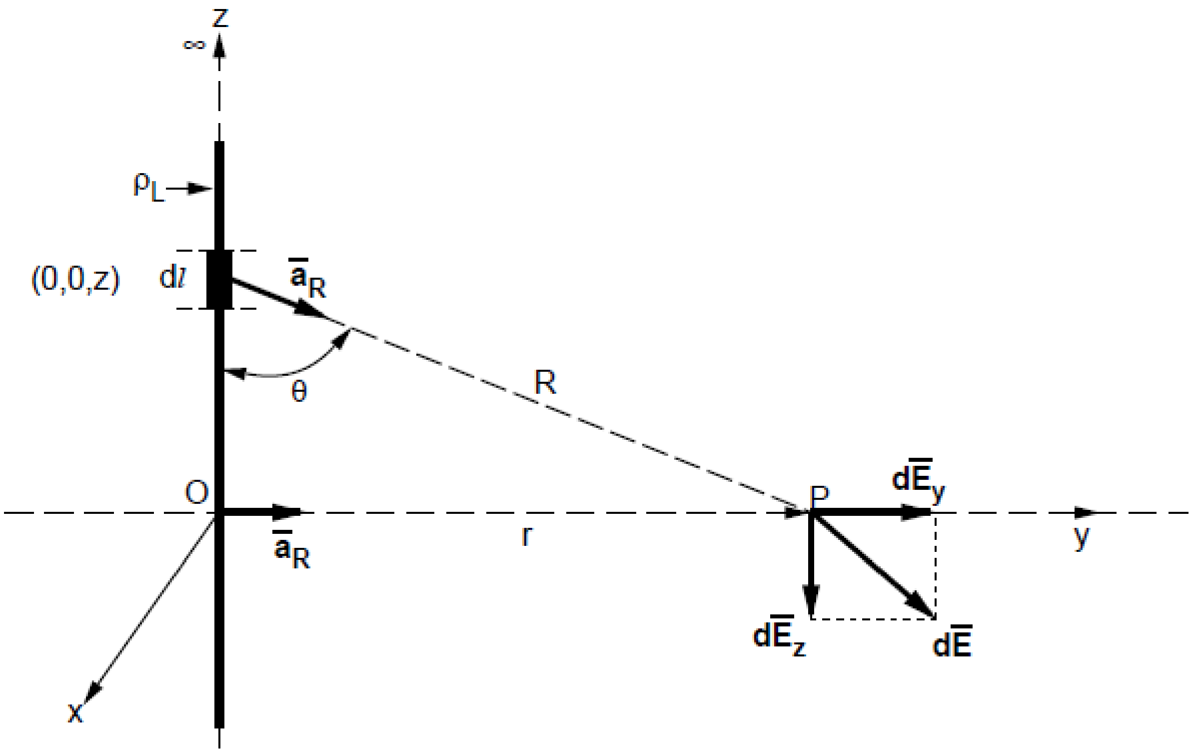
ELECTROMAGNETIC FIELDS AND WAVES

Electric Field Due to an Line Charge



A **line charge** is a continuous charge distribution along a thin wire or filament.

The charge is characterized by the **linear charge density** λ.

$$\lambda = \frac{dq}{dl} \quad (\mathrm{C/m})$$

$\lambda \rightarrow$ Linear charge density (C/m)

dq $\rightarrow$ Small charge element (C)

dl$\rightarrow$ Small length element (m) dl=dz

$$dQ = \rho_L \, dl = \rho_L \, dz$$

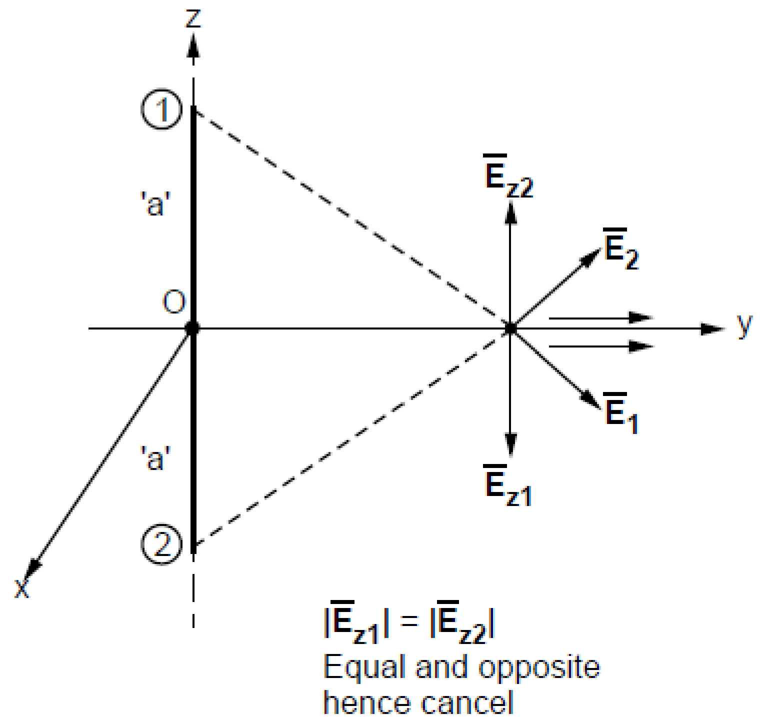
$$\bar{R} = \bar{r}_P - \bar{r}_{dl} = \left[r\,\bar{a}_y - z\,\bar{a}_z \right]$$

$$\therefore \; |\bar{R}| = \sqrt{r^2 + z^2}$$

$$\therefore \; \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{r\,\bar{a}_y - z\,\bar{a}_z}{\sqrt{r^2+z^2}}$$

$$\therefore \; d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R =$$

$$\frac{\rho_L \, dz}{4\pi\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^2} \left[\frac{r\,\bar{a}_y - z\,\bar{a}_z}{\sqrt{r^2+z^2}} \right] \quad \ldots$$



$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{r\sec^3\theta} \bar{a}_y \quad \Big\} \text{ changing the limits}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} [\sin\theta]_{-\pi/2}^{\pi/2} \bar{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r} \left[\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right] \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} [1-(-1)] \bar{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r} \times 2 \, \bar{a}_y$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_y \quad V/m}$$


Thank you