Module - II

Simplex Method - 1

To solve a linear programming problem graphical method is used when only two decision variable are present. But ~~when~~ most real life problems when formulated as LP model will have more than two decision variables. A "Simplex Method" is used for solving LPP with a larger number of variables.

The Essence of Simplex method

Simplex method is an algebraic procedure. Its underlying concepts are geometric.

The geometric concepts are related to the algebra of the Simplex method. In graphical method of solving an LPP, we used to identify a common region known as feasible region satisfying all the constraints. The optimal solution used to occur at some vertex of the feasible region.

If the optimal solution was not unique, the optimal points were on an edge. Essentially the problem is that of finding the particular vertex for the convex region which corresponds to the optimal solution.

The Simplex method provides an algorithm which consists in moving from one vertex of the region of feasible solution to another vertex in such a way that the value of the objective function at the succeeding vertex is improved than at the previous vertex

This procedure of iteration from one vertex to another is repeated till the optimal solution is obtained. Thus, the geometric concepts are related to the algebra on which simplex method works.

BASIC TERMS / DEFINITIONS

Slack Variable: A variable added to the left hand side of a constraint (less than or equal to) to convert the constraint into an equation is called a slack variable.

Eg: If the constraint given is $3x_1+5x_2 \leq 10$ then
$3x_1 + 5x_2 + S_1 = 10$ is the equality (equation)
where S_1 is a slack variable.

Surplus Variable: A variable subtracted from the left hand side of the constraint (greater than or equal to) and to convert it into an equality is called a surplus variable.

Eg: If the constraint given is $5x_1+8x_2 \geq 12$ then
$5x_1+8x_2-S_1 = 12$ is the equation.
where S_1 is the surplus variable.

Basic Solution: The initial solution obtained after setting the basic variables as zeros is basic solution. It is the unique solution resulting from setting $(n-m)$ variables equal to zero.

Where,
m = number of simultaneous linear equations
n = number of variables.

Basic feasible solution: A basic solution which satisfies $x_i \geq 0$, $i = 1, 2, \ldots n$ is called a basic feasib solution.

Optimal Solution: Any basic feasible solution which optimizes (minimizes or maximizes) the objective functi of a general LP problem is known as an optimal solution.

Standard form of an LP problem (Characteristics of LPF

The standard form of the LP problem should have t following characteristics.

(i) All the constraints should be expressed as equa by adding slack or surplus variables.

(ii) The right hand side of each constraint should be made non negative. if it is not, this should be done by multiplying both sides of the resultin constraint by -1.

(iii) The objective function should be of the maximizati type (if it is not, should be converted by multiplying with -1).

(eg) obtain all the basic solutions to the following system of linear equations. Is the non-degenerate solution feasible?

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

which of them are basic feasible solutions.

Soln:

Number of unknown (variables) $= 4$
Number of equations $= 2$
There will be $4C_2$ $(=6)$ different possible basic solutions.

S.No	Basic Variable	Non-Basic Variable	Value of basic Variables	Is the solution feasible.
1	x_1, x_2	$x_3 = x_4 = 0$	$2x_1 + 6x_2 = 3$ $6x_1 + 4x_2 = 2$ $x_1 = 0$, $x_2 = \frac{1}{2}$	Yes
2	x_1, x_3	$x_2 = x_4 = 0$	$2x_1 + 2x_3 = 3$ $6x_1 + 4x_3 = 2$ $x_1 = -2$, $x_3 = 7/2$	No
3	x_1, x_4	$x_2 = x_3 = 0$	$2x_1 + x_4 = 3$ $6x_1 + 6x_4 = 2$ $x_1 = 8/4$, $x_4 = -7/3$	NO
4	x_2, x_3	$x_1 = x_4 = 0$	$6x_2 + 2x_3 = 3$ $4x_2 + 4x_3 = 2$ $x_2 = \frac{1}{2}$, $x_3 = 0$	Yes
5	x_2, x_4	$x_1 = x_3 = 0$	$6x_2 + x_4 = 3$ $4x_2 + 6x_4 = 4$ $x_2 = \frac{1}{2}$, $x_4 = 0$	Yes
6	x_3, x_4	$x_1 = x_2 = 0$	$2x_3 + x_4 = 3$ $4x_3 + 6x_4 = 2$ $x_3 = 2$, $x_4 = -1$	NO

Eg Find all the basic solutions of the following system of equations identifying in each case the basic & nonbasic variables

$$2x_1 + x_2 + 4x_3 = 1$$
$$3x_1 + x_2 + 5x_3 = 14$$

Soln. No. of equations = 2
No. of variables = 3
$\therefore$ There are $3C_2$ possible ways of getting different basic solutions $3C_2 = \frac{3 * 2}{2} = 3$ ways.

When, $x_3 = 0$, $2x_1 + x_2 = 11$, $3x_1 + x_2 = 14$
on solving $x_1 = 3$, $x_2 = 5$

$x_1 = 0$, $x_2 + 4x_3 = 11$, $x_2 + 5x_3 = 14$
on solving $x_2 = -1$, $x_3 = 3$

$x_2 = 0$, $2x_1 + 4x_3 = 11$, $3x_1 + 5x_3 = 14$
on solving $x_1 = 1/2$, $x_3 = 5/2$

S.No	Basic Variables	Non-basic Variables	Value of basic Variables	Is the soln feasible.
1	x_1, x_2	x_3	$x_1 = 3$, $x_2 = 5$	Yes
2	x_2, x_3	x_1	$x_2 =$ [illegible], $x_3 =$ [illegible]	No
3	x_1, x_3	x_2	$x_1 = 1/2$, $x_3 = 5/2$	Yes

Infeasible solution: If the basic variable values are negative the solution is stated as infeasible.