UNIT – I PROPERTIES OF MATTER

TOPIC – V CANTILEVER

2.15 CANTILEVER

A cantilever is a beam fixed horizontally at one end and loaded at the other end.

Theory:

Let us consider a beam fixed at one-end and loaded at its other end as shown in Fig 2.15. Due to the load applied at the free end, a couple is created between the two forces. (i.e)

(i) Force (load 'W') applied at the free end towards downward direction and

(ii) Reaction (R) acting in the upward direction at the supporting end.

This external bending couple tends to bend the beam in the clockwise direction. But, since one end of the beam is fixed, the beam cannot rotate. Therefore the external bending couple must be balanced by another equal and opposite couple, created due to the elastic nature of the body (i.e) called as internal bending moment.

Under equilibrium condition,

External bending moment = Internal bending moment

2.16 DEPRESSION OF A CANTILEVER – LOADED AT ITS ENDS

Theory

Let 'l' be the length of the cantilever OA fixed at 'O'. Let 'W' be the weight suspended (loaded) at the free end of the cantilever. Due to the load applied the cantilever moves to a new position OA' as shown in Fig 2.16.

Let us consider an element PQ of the beam of length dx, at a distance OP= x from the fixed end. Let 'C' be the centre of curvature of the element PQ and let 'R' be the radius of curvature.

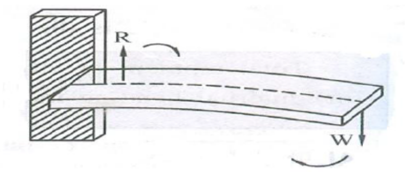

Fig 2.15

Due to the load applied at the free end of the cantilever, an external couple is created between the load W at 'A' and the force of reaction at 'Q'. Here the arm of the couple (Distance between the two equal and opposite forces) is (l-x).

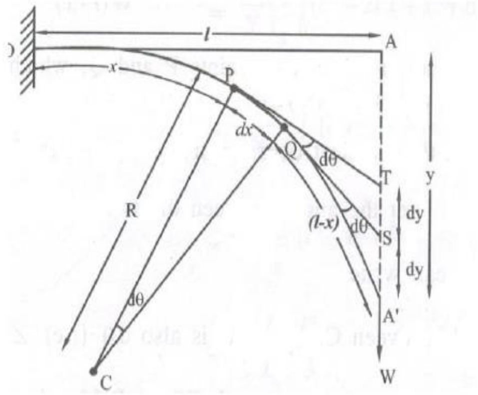

Fig 2.16

The external bending moment = W. (l-x) -------------- (1)

We know the internal bending moment = $\frac{YI_g}{R}$ ------------- (2)

We know under equilibrium condition

External bending moment = Internal bending moment

Therefore, we can write Eqn. (1) = Eqn. (2)

(i.e) W. (l-x) = $\frac{YI_g}{R}$

(i.e) $R = \frac{YI_g}{W.(l-x)}$ ---------------(3)

Two tangents are drawn at points P and Q, which meet the vertical line AA' at T and S respectively.

Let the smallest depression produced from T and S = dy

And Let the angle between the two tangents = dθ

Then we can write

The angle between CP and CQ is also dθ (i.e) $< PCQ$ = dθ

We can write the length PQ = R dθ = dx

(or) $d\theta = \frac{dx}{R}$ ---------------(4)

Substituting eqn. (3) in eqn. (4), we have $d\theta = \frac{dx}{\frac{YI_g}{W.(l-x)}}$

From the ΔQA'S we can write $\sin d\theta = \frac{dy}{(l-x)}$

(or) $$d\theta = \frac{W}{YI_g}(l-x)\,.\,dx \qquad \text{-----------------(5)}$$

If dθ is very small then we can write,

$$dy = (l-x)\,d\theta \qquad \text{-----------------(6)}$$

Substituting eqn. (5) in eqn. (6) we have

$$dy = \frac{W}{YI_g}(l-x)^2\,.\,dx \qquad \text{-----------------(7)}$$

Total depression at the free end of the cantilever can be derived by integrating the eqn. (7) within the limits 0 to 'l'.

$$y = \frac{W}{YI_g}\int_0^l (l-x)^2\,.\,dx$$

$$= \frac{W}{YI_g}\int_0^l (l^2 - 2lx + x^2)\,.dx$$

$$= \frac{W}{YI_g}\int_0^l \left(l^2x - \frac{2lx^2}{2} + \frac{x^3}{3}\right)$$

$$= \frac{W}{YI_g}\left[l^3 - l^3 + \frac{l^3}{3}\right]$$

$$y = \frac{W}{YI_g}\,.\,\frac{l^3}{3} = \frac{Wl^3}{3YI_g} \qquad \text{------------------(8)}$$

Special cases

(i) Rectangular cross section

If 'b' is the breadth and 'd' is the thickness of the beam then we know

$$I_g = \frac{bd^3}{12}$$

Substituting the value of I_g in eqn .(8), we can write

The depression produced at free end for a rectangular cross section

$$y = \frac{Wl^3}{3Y(bd^3/12)}$$

$$y = \frac{4Wl^3}{Ybd^3}$$

(ii) Circular cross section

If 'r' is the radius of the circular cross section, then

We know $$I_g = \frac{\pi r^4}{4}$$

Substituting the value of I_g in eqn. (8), we can write

Depression produced $$y = \frac{Wl^3}{3Y(\frac{\pi r^4}{4})} \text{ (or) } y = \frac{4Wl^3}{3\pi r^4 Y}$$