



SNS COLLEGE OF ENGINEERING

An Autonomous Institution

Coimbatore- 107



Accredited by NAAC – UGC with 'A' Grade Approved by AICTE, New Delhi &
Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECB203 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 2 – IIR FILTER DESIGN

TOPIC – CHEBYSHEV FILTER

3/12/2025



COMPARISON OF DIGITAL & ANALOG FILTERS



S.No.	Digital Filter	Analog Filter
1	Operates on digital samples of the signal	Operates on analog signals
2	It is governed by linear difference equation	It is governed by linear differential equation
3	It consists of adders, multipliers and delays implemented in digital logic	It consists of electrical components like resistors, capacitors and inductors
4	The filter coefficients are designed to satisfy the desired frequency response	The approximation problem is solved to satisfy the desired frequency response



DESIGN OF LOWPASS DIGITAL CHEBYSHEV FILTER



- For designing a Chebyshev IIR digital filter, analog filter is designed using the given specifications
- Then the analog filter transfer function is transformed to digital filter transfer function by using either Impulse Invariant or Bilinear Transformation
- The analog chebyshev filter is designed by approximating the ideal frequency response using an error function
- The approximation function is selected such that the error is minimized over a band of frequencies



TYPES OF CHEBYSHEV APPROXIMATION



- There are two types of Chebyshev approximation:
 1. Type-1 Chebyshev Approximation
 2. Type-2 Chebyshev Approximation
- In type - 1 approximation, the error function is selected such that magnitude response is equiripple in the passband and monotonic in the stopband
- In type - 2 approximation, the error function is selected such that magnitude response is monotonic in the passband and equiripple in the stopband
- The type – 2 magnitude response is also called Inverse chebyshev response



ANALOG CHEBYSHEV FILTER



- The magnitude response of Type-1 low pass filter is given by

$$\left| H(j\Omega) \right|^2 = \frac{1}{1 + \epsilon^2 C_N^2 \left(\frac{\Omega}{\Omega_p} \right)}$$

- ϵ – Attenuation Constant

$$C_N \left(\frac{\Omega}{\Omega_p} \right) = \text{Chebyshev polynomial of order } N$$



ANALOG CHEBYSHEV FILTER



- Attenuation Constant is given by

$$\epsilon = \left[\frac{1}{A_p^2} - 1 \right]^{1/2}$$

- A_p - is the gain or magnitude at pass band edge frequency Ω_p
- For small values of N the chebyshev polynomial is given by

$$C_N(x) = \begin{cases} \cos(N \cos^{-1} x) & ; \text{ for } |x| \leq 1 \\ \cosh(N \cosh^{-1} x) & ; \text{ for } |x| > 1 \end{cases}$$



ANALOG CHEBYSHEV FILTER



- The transfer function of the analog system can be obtained from the magnitude response of Type – 1 low pass filter by substituting Ω by s/j

$$H(s) H(-s) = \frac{1}{1 + \epsilon^2 C_N^2 \left(\frac{s/j}{\Omega_c} \right)}$$

- For the normalized transfer function, let us replace s/Ω_c by s_n

$$H(s_n) H(-s_n) = \frac{1}{1 + \epsilon^2 C_N^2 (-js_n)}$$



PROPERTIES OF CHEBYSHEV FILTERS (TYPE -1)



- The magnitude $|H(j\Omega)|$ oscillates between 1 and $1/\sqrt{1+\epsilon^2}$ within the pass band and so the filter is called equiripple in the pass band
- The normalized magnitude response has a value of $1/\sqrt{1+\epsilon^2}$ at cutoff frequency Ω_c
- The magnitude is monotonic outside the pass band
- The Chebyshev Type – 1 Filters are all pole designs
- With large values of N , the transition from pass band to stop band becomes more sharp and approaches ideal characteristics.



TRANSFER FUNCTION OF ANALOG CHEBYSHEV LOW PASS FILTER



- For a stable and causal filter the poles should lie on the left half of s -plane.
Hence the desired filter transfer function is obtained by selecting N number of left half poles
- When N is even all the poles are complex and exist as conjugate pair
- When N is odd, one of the pole is real and all other poles are complex and exist as conjugate pair
- Therefore the transfer function of chebyshev filters will be a product of second order factors



NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



- N be the order of the filter
- $H(s_n)$ be the normalized Chebyshev lowpass filter function
- When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

- When N is odd,

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$



NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



$$\text{where, } b_k = 2 y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2} \left\{ \left[\left(\frac{1}{\epsilon^2} + 1 \right)^{\frac{1}{2}} + \frac{1}{\epsilon} \right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2} + 1 \right)^{\frac{1}{2}} + \frac{1}{\epsilon} \right]^{-\frac{1}{N}} \right\}$$



NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



- N be the order of the filter
- For even values of N parameter B_k are evaluated

$$H(s_n)|_{s_n = 0} = \frac{1}{(1 + \epsilon^2)^{\frac{1}{2}}}$$

- For odd values of N the parameter B_k are evaluated

$$H(s_n)|_{s_n = 0} = 1$$

- While evaluating B_k to take $B_0 = B_1 = B_2 = \dots = B_k$



UNNORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



- The unnormalized transfer function is obtained by letting $s_n \rightarrow s / \Omega_c$ in the normalized transfer function, Where Ω_c is the cutoff frequency of the lowpass filter
- $H(s)$ be the normalized Chebyshev low pass filter transfer function
- When N is even, $H(s)$ is obtained by letting $s_n \rightarrow s / \Omega_c$ in normalized Chebyshev low pass filter function

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \bigg|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$



UNNORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



- $H(s)$ be the normalized Chebyshev low pass filter transfer function
- N be the order of the filter
- Ω_c is the cutoff frequency of the lowpass filter
- When N is odd, $H(s)$ is obtained by letting $s_n \rightarrow s / \Omega_c$ in normalized Chebyshev low pass filter function

$$\therefore H(s) = \frac{B_0}{s_n + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$



ORDER OF ANALOG LOWPASS CHEBYSHEV FILTER



- In Chebyshev filters the frequency response of the filter depends on the order N. The specifications of the filter are given in terms of gain at a passband and stopband frequency
- A_p - Gain or Magnitude at pass band edge frequency Ω_p
- A_s - Gain or Magnitude at Stop band edge frequency Ω_s

$$N_1 = \frac{\cosh^{-1} \left[\left(\frac{(1/A_s^2) - 1}{(1/A_p^2) - 1} \right)^{\frac{1}{2}} \right]}{\cosh^{-1} \left(\frac{\Omega_s}{\Omega_p} \right)}$$



ORDER OF THE LOWPASS CHEBYSHEV FILTER

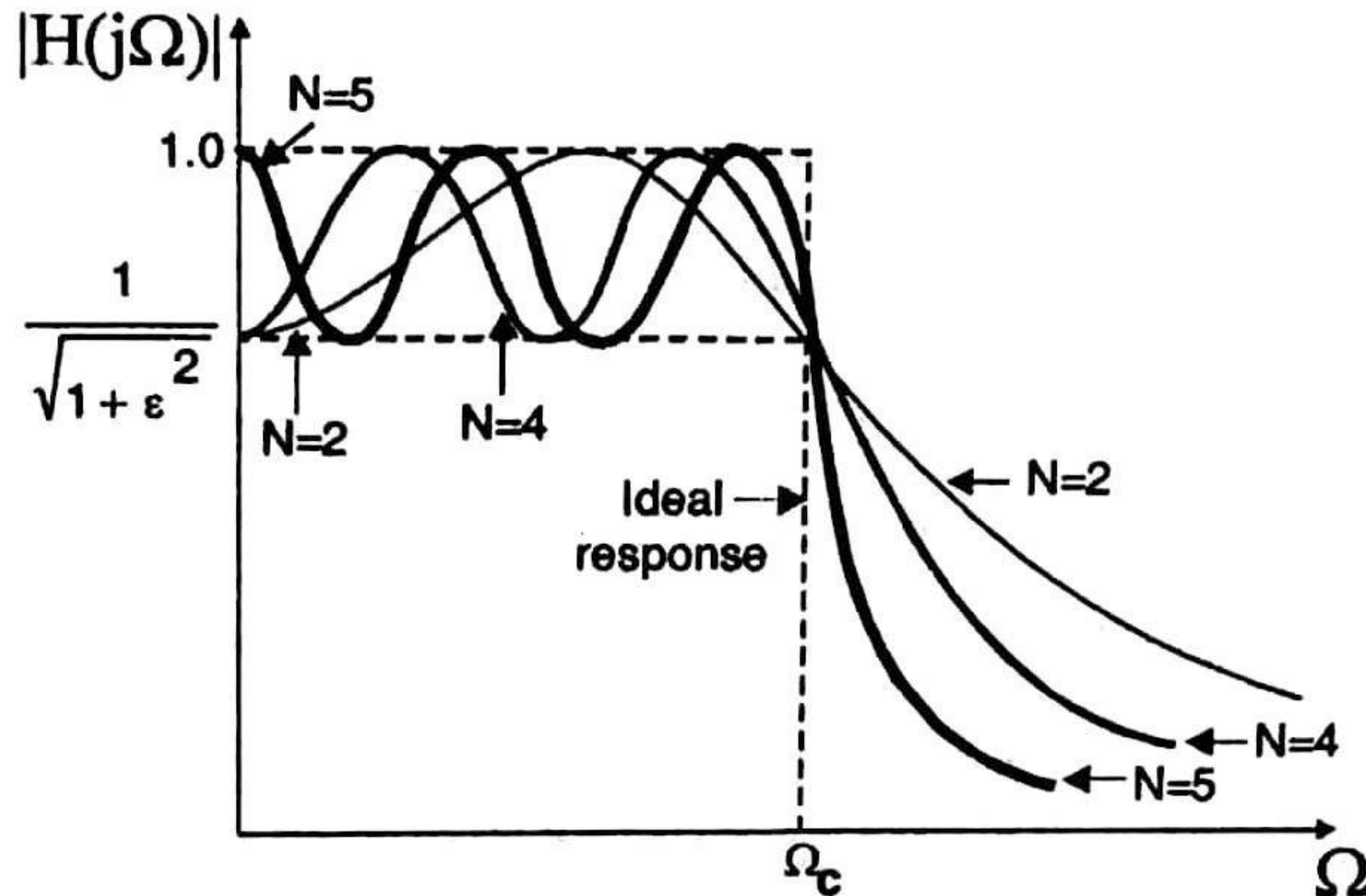


- The specifications of the filter are given in terms of dB attenuation at a passband and stopband frequency
- $\alpha_{p, \text{dB}}$ - dB attenuation at pass band edge frequency Ω_p
- $\alpha_{s, \text{dB}}$ - dB attenuation at Stop band edge frequency Ω_s

$$N_1 = \frac{\cosh^{-1} \left[\left(\frac{10^{0.1\alpha_{s, \text{dB}}} - 1}{10^{0.1\alpha_{p, \text{dB}}} - 1} \right)^{\frac{1}{2}} \right]}{\cosh^{-1} \left(\frac{\Omega_s}{\Omega_p} \right)}$$



MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 1 LOW PASS FILTER

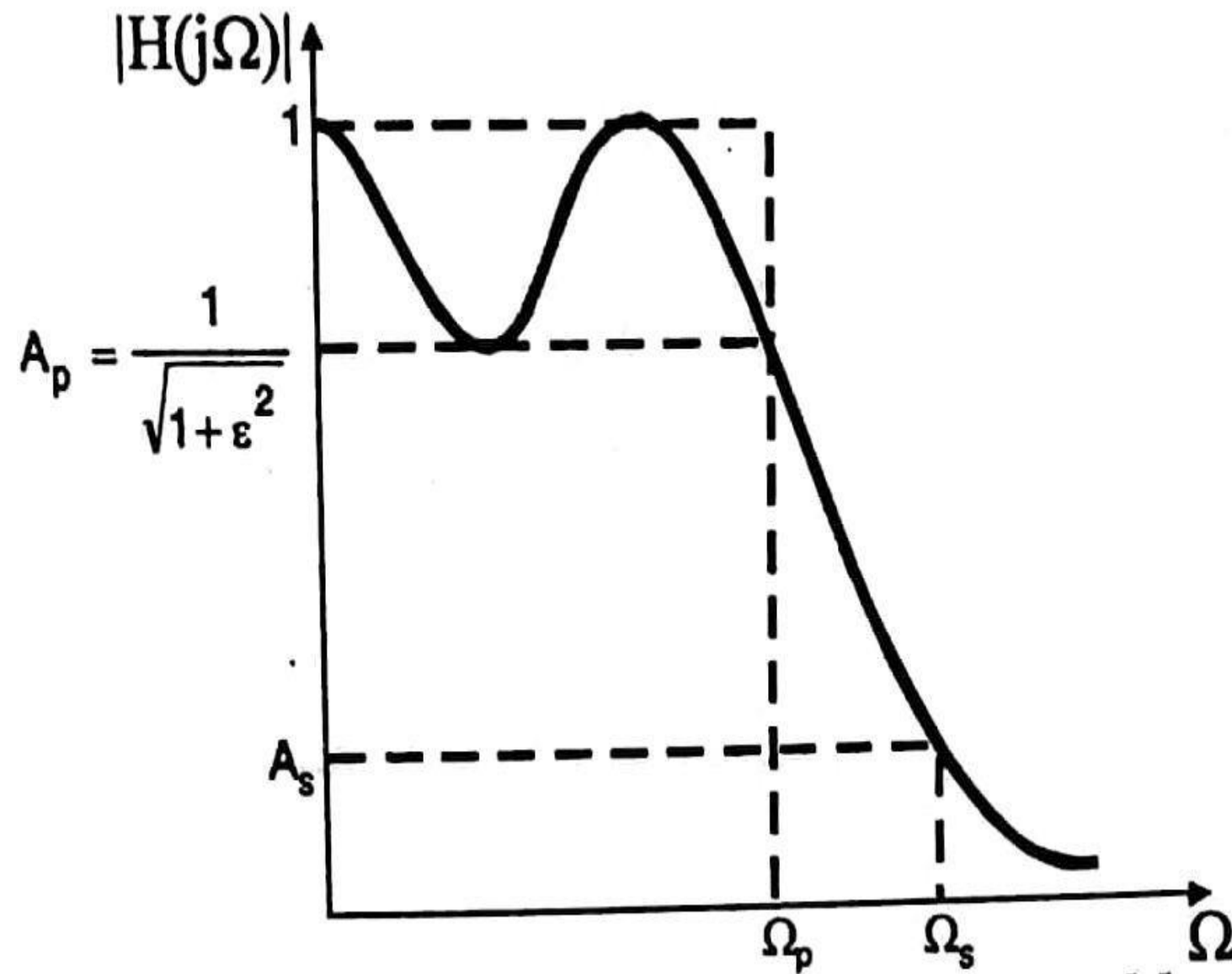




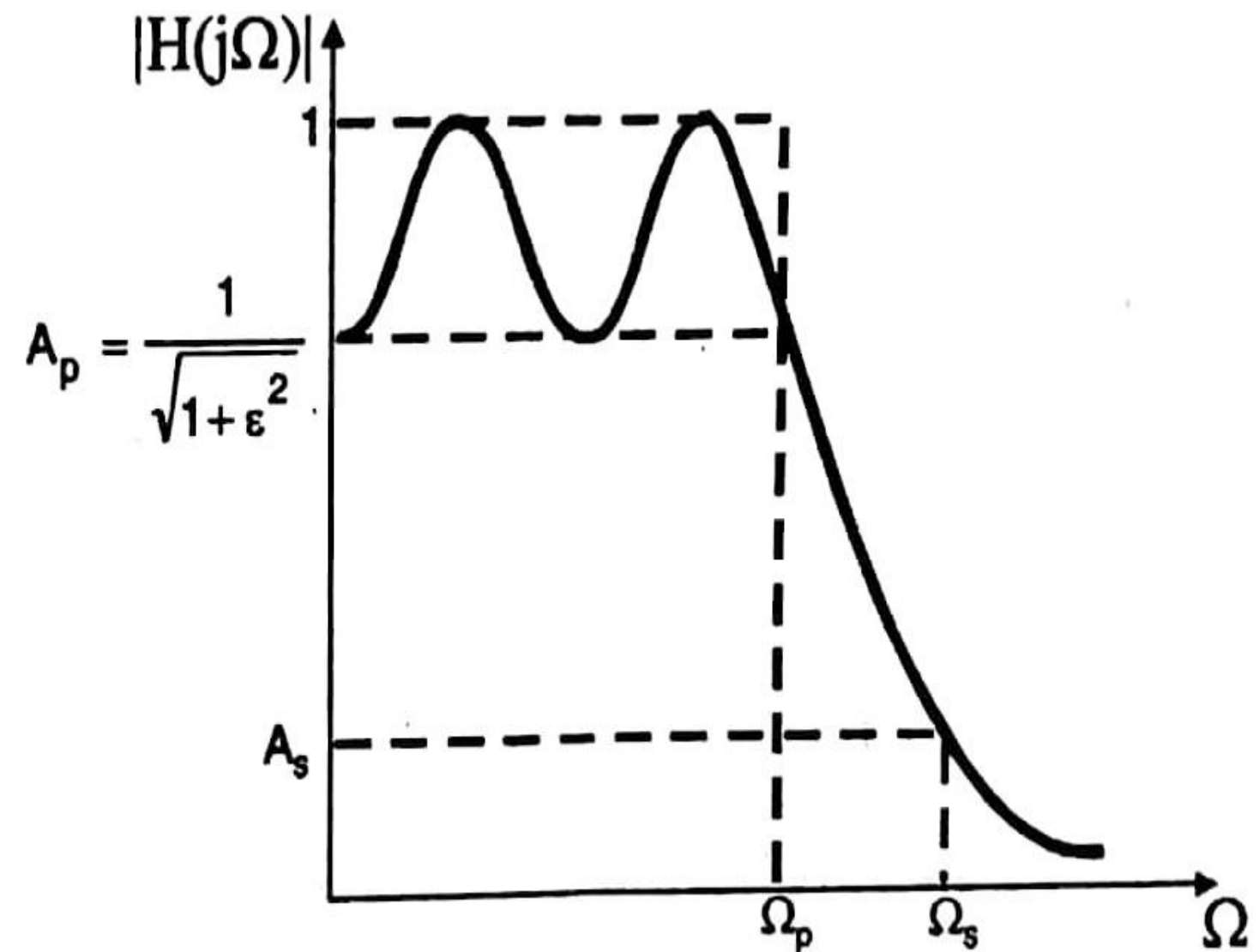
MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 1 FILTERS



N is Odd



N is Even

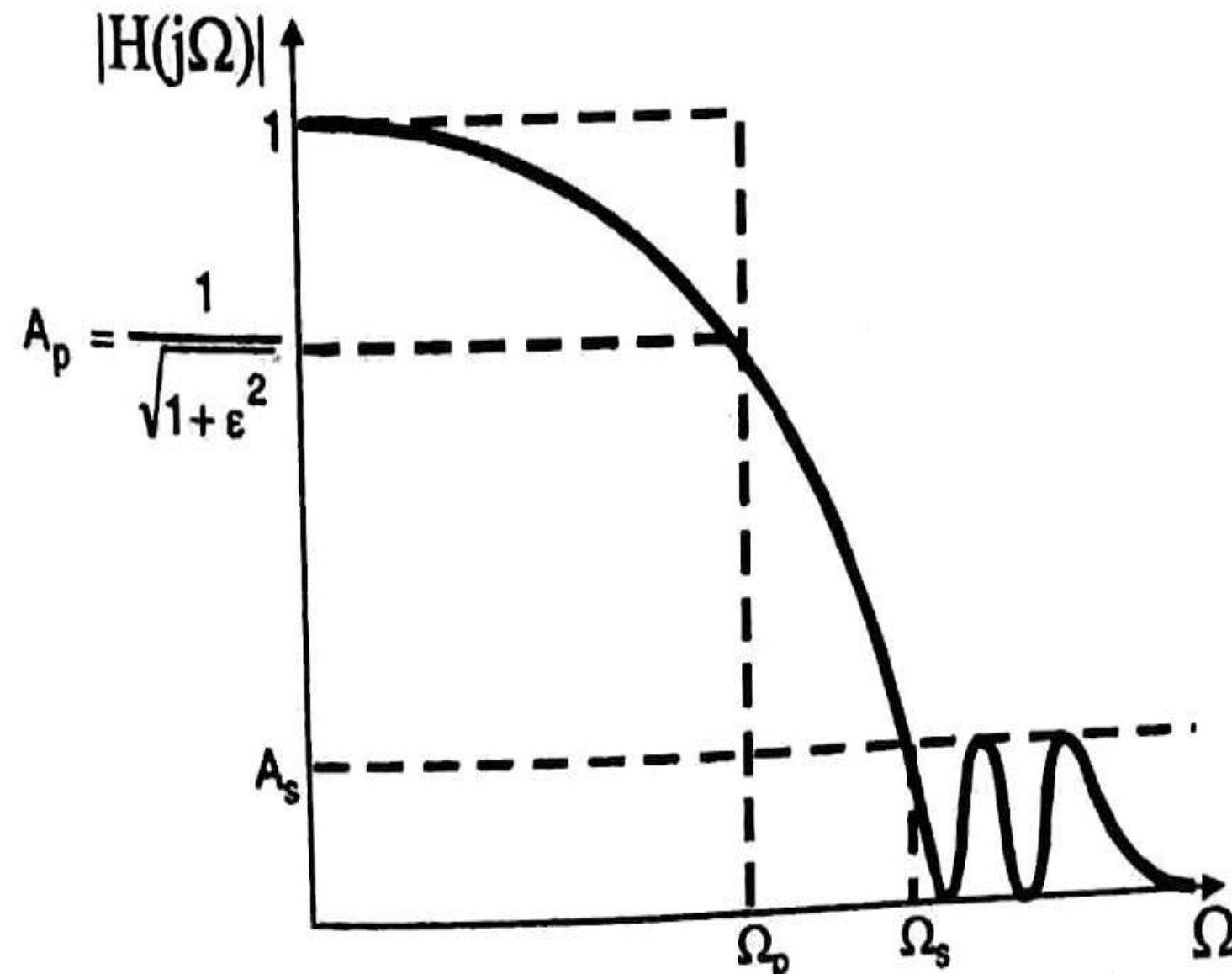




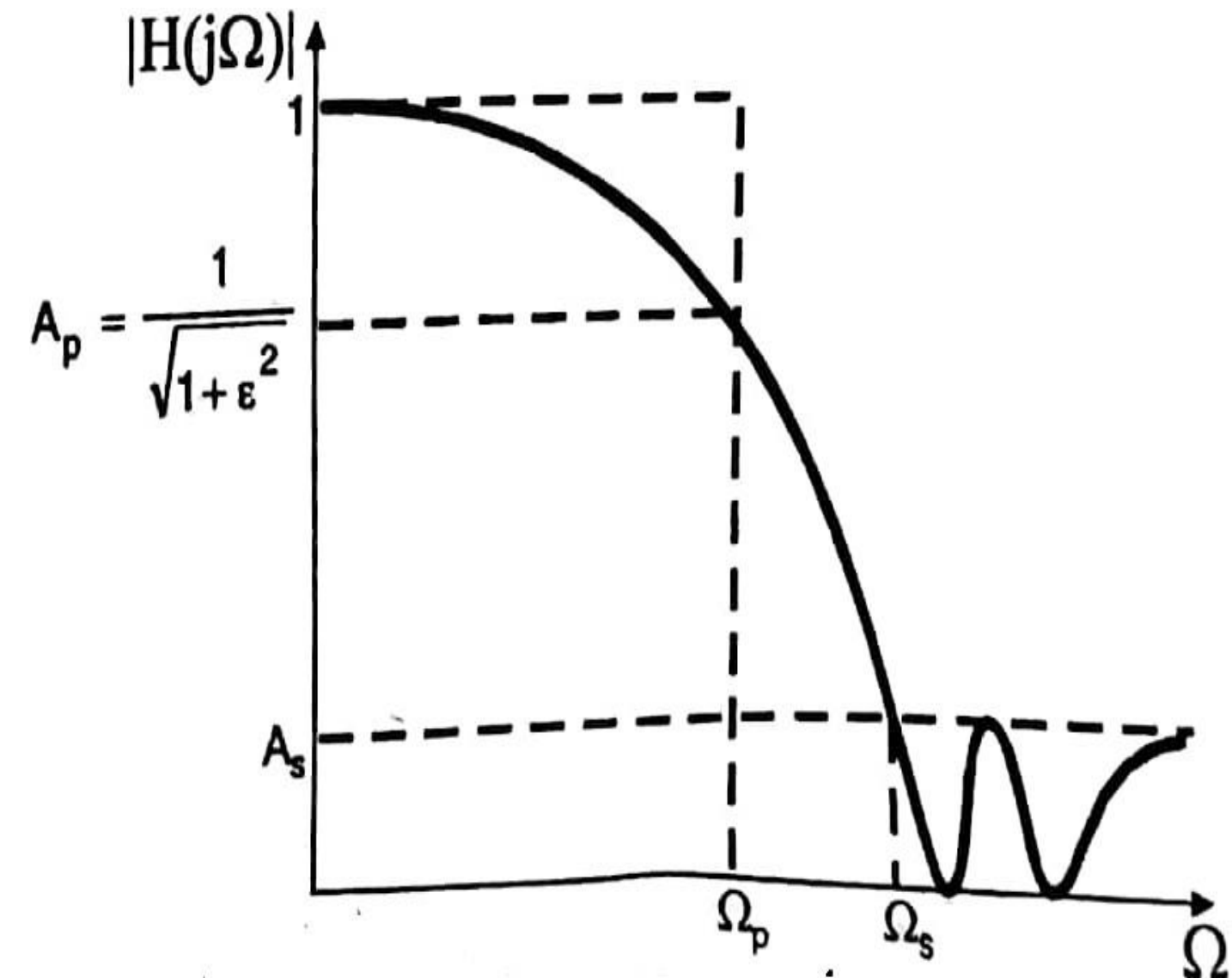
MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 2 FILTERS



N is Odd



N is Even





CUTOFF FREQUENCY OF ANALOG LOWPASS CHEBYSHEV FILTER



- The IIR filters are designed to satisfy a prescribed gain or attenuation at a pass band or stop band frequency. But Practically the cutoff frequency Ω_c is used to decide the useful frequency range of the filter
- In chebyshev filter design the passband and stopband specifications are used to estimate the order, N of the filter and N^{th} order normalized Chebyshev lowpass filter is designed. Then the normalized LPF is unnormalized using the cutoff frequency
- In Chebyshev filters the passband edge frequency, Ω_p is considered as cutoff frequency Ω_c and this cutoff is not equal to 3 dB cutoff frequency $\Omega_{3\text{dB}}$



DESIGN PROCEDURE FOR LOWPASS DIGITAL CHEBYSHEV IIR FILTER



- ω_p - Pass band edge digital frequency in rad /sample
- ω_s - Stop band edge digital frequency in rad/sample
- A_p - Gain at pass band edge frequency ω_p
- A_s - Gain at Stop band edge frequency ω_s
- $T = 1 / F_s$ - Sampling time in sec.
- Where F_s = Sampling frequency in Hz
- Ω_p - Pass band edge analog frequency corresponding to ω_p
- Ω_s - Stop band edge analog frequency corresponding to ω_s



DESIGN PROCEDURE FOR LOWPASS DIGITAL CHEBYSHEV IIR FILTER



1. Choose either Bilinear or Impulse Invariant transformation and determine the specifications of equivalent analog filter

- The gain or attenuation of analog filter is same as digital filter

- **Bilinear Transformation:**

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2} \quad \Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2}$$

- **Impulse Invariant Transformation:**

$$\Omega_p = \frac{\omega_p}{T} \quad \Omega_s = \frac{\omega_s}{T}$$



ORDER OF THE LOWPASS DIGITAL CHEBYSHEV FILTER



2. Decide the order N of the filter. In order to estimate the order N , Calculate the Parameter N_1 using the following equation:

$$N_1 = \frac{\cosh^{-1} \left[\left(\frac{(1/A_s^2) - 1}{(1/A_p^2) - 1} \right)^{\frac{1}{2}} \right]}{\cosh^{-1} \left(\frac{\Omega_s}{\Omega_p} \right)}$$

- Choose N such that, $N \geq N_1$, Usually N is chosen as nearest integer just greater than N_1



NORMALIZED BUTTERWORTH LPF TRANSFER FUNCTION



3. Determine the normalized transfer function $H(s_n)$ of the analog lowpass filter function

- When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

- When N is odd,

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$



NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



$$\text{where, } b_k = 2 y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right)$$

$$\epsilon = \left[\frac{1}{A_p^2} - 1 \right]^{1/2}$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2} \left\{ \left[\left(\frac{1}{\epsilon^2} + 1 \right)^{1/2} + \frac{1}{\epsilon} \right]^{1/N} - \left[\left(\frac{1}{\epsilon^2} + 1 \right)^{1/2} + \frac{1}{\epsilon} \right]^{-1/N} \right\}$$



NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION



- N be the order of the filter
- For even values of N parameter B_k are evaluated
- For odd values of N the parameter B_k are evaluated
- While evaluating B_k to take $B_0 = B_1 = B_2 = \dots = B_k$



UNNORMALIZED ANALOG TRANSFER FUNCTION



4. Determine the unnormalized analog transfer function $H(s)$ is obtained by replacing s_n by s/Ω_c in the normalized transfer function of the low pass filter function

- When N is even,

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \bigg|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

- When N is odd,

$$\therefore H(s) = \frac{B_0}{s_n + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$



DESIGN PROCEDURE FOR LOWPASS DIGITAL CHEBYSHEV IIR FILTER



5. Determine the transfer function of digital filter $H(z)$. Using the suitable transformation to transform $H(s)$ to $H(z)$. When the Impulse invariant transformation is employed, if $T < 1$, then multiply $H(z)$ by T to normalize the magnitude.
6. Realize the digital filter transfer function $H(z)$ by a suitable structure
7. Verify the design by sketching the frequency response $H(e^{j\omega})$

$$H(e^{j\omega}) = H(z) / z = e^{j\omega}$$



ASSESSMENT



1. What is Chebyshev approximation?
2. How will you choose the order N for a Chebyshev Filter?
3. List the Properties of Chebyshev Filter.
4. Compare Analog filter and Digital filter?
5. Analog filter transfer function is converted to a digital filter transfer function by using either ----- (or) -----
6. Define Sampling Time.
7. Attenuation Constant is given by -----
8. List the types of Chebyshev filters.



THANK YOU