UNIT – II LASER AND FIBER OPTICS

TOPIC – II: Einstein's A and B coefficients -derivation.

Einstein Coefficient Relation derivation and discussion:

Einstein showed the interaction of radiation with matter with the help of three processes called stimulated absorption, spontaneous emission and stimulated emission. He showed in 1917 that for proper description of radiation with matter, the process of stimulated emission is essential. Let us first derive the Einstein coefficient relation on the basis of above theory:

Let N_1 be the number of atoms per unit volume in the ground state E_1 and these atoms exist in the radiation field of photons of energy E_2-E_1 =h v such that energy density of the field is E.

Let R_1 be the rate of absorption of light by E_1 -> E_2 transitions by the process called stimulated absorption

This rate of absorption R_1 is proportional to the number of atoms N_1 per unit volume in the ground state and proportional to the energy density E of radiations.

That is $R_1 \infty N_1 E$

Or $R_1 = B_{12} N_1 E$ (1)

Where B_{12} is known as the Einstein's coefficient of stimulated absorption and it represents the probability of absorption of radiation. Energy density e is defined as the incident energy on an atom as per unit volume in a state.

Now atoms in the higher energy level E_2 can fall to the ground state E_1 automatically after 10^{-8} sec by the process called spontaneous emission.

The rate R_2 of spontaneous emission E_2-> E_1 is independent of energy density E of the radiation field.

R_2 is proportional to number of atoms N_2 in the excited state E_2 thus

$R_2 \infty N_2$

$R_2 = A_{21} N_2$ (2)

Where A_{21} is known as Einstein's coefficient for spontaneous emission and it represents the probability of spontaneous emission.

Atoms can also fall back to the ground state E_1 under the influence of electromagnetic field of incident photon of energy E_2-E_1 =hv by the process called stimulated emission

Rate R_3 for stimulated emission E_2-> E_1 is proportional to energy density E of the radiation field and proportional to the number of atoms N_2 in the excited state, thus

$R_3 \alpha N_2 E$

Or $R_3=B_{21}N_2 E$ (3)

Where B_{21} is known as the Einstein coefficient for stimulated emission and it represents the probability of stimulated emission.

In steady state (at thermal equilibrium), the two emission rates (spontaneous and stimulated) must balance the rate of absorption.

Thus $R_1=R_2+R_3$

Using equations (1,2, and 3) ,we get

$N_1B_{12}E=N_2A_{21}+N_2B_{21}E$

Or $N_1B_{12}E - N_2B_{21}E=N_2A_{21}$

Or $(N_1B_{12}-N_2B_{21}) E = N_2A_{21}$

Or $E= N_2A_{21}/N_1B_{12}-N_2B_{21}$

$= N_2A_{21}/N_2B_{21}[N_1B_{12}/N_2B_{21} -1]$

[by taking out common $N_2B_{21}$from the denominator]

Or $E=A_{21}/B_{21} \{1/N_1/N_2(B_{12}/B_{21}-1))$ (4)

Einstein proved thermodynamically, that the probability of stimulated absorption is equal to the probability of stimulated emission.

$B_{12}=B_{21}$

Then equation (4) becomes

$E=A_{21}/B_{21} (1/N_1/N_2-1)$ (5)

From Boltzman's distribution law, the ratio of populations of two levels at temperature T is expressed as

$N_1/N_2=e^{(E_2-E_1)/KT}$

$N_1/N_2=e^{hv/KT}$

Where K is the Boltzman's constant and h is the Planck's constant.

Substituting value of N_1/N_2in equation (5) we get

$E= A_{21}/B_{21}(1/e^{hv/KT}-1)$ (6)

Now according to Planck's radiation law, the energy density of the black body radiation of frequency v at temperature T is given as

$E = 8\pi hv^3/c^3(1/e^{hv/KT})$ (7)

By comparing equations (6 and 7),we get

$A_{21}/B_{21}=8\pi hv^3/c^3$

This is the relation between Einstein's coefficients in laser.