SNS COLLEGE OF ENGINEERING

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DEPARTMENT OF COMPUTER SCIENCE AND DESIGN

COURSE NAME : 19EE01 BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

I YEAR /II SEMESTER

Unit 5 – LINEAR AND DIGITAL ELECTRONICS

Topic 4 : Digital: Boolean Algebra-Theorems

Boolean postulates and basic laws that are used in Boolean algebra. These are useful in minimizing Boolean functions.

Boolean Postulates

Consider the binary numbers 0 and 1, Boolean variable *x* and its complement *x*′.

Either the Boolean variable or complement of it is known as **literal**. The four possible **logical OR** operations among these literals and binary numbers are shown below.

Similarly, the four possible **logical AND** operations among those literals and binary numbers are shown below.

$x + 0 = x$	$x.1 = x$
$x + 1 = 1$	$x.0 = 0$
$x + x = x$	$x.x = x$
$x + x' = 1$	$x.x' = 0$

These are the simple Boolean postulates. We can verify these postulates easily, by substituting the Boolean variable with ‘0’ or ‘1’.

Note− The complement of complement of any Boolean variable is equal to the variable itself. i.e., $x''=x$.

Basic Laws of Boolean Algebra

Following are the three basic laws of Boolean Algebra.

- Commutative law
- Associative law
- Distributive law

Commutative Law

If any logical operation of two Boolean variables give the same result irrespective of the order of those two variables, then that logical operation is said to be **Commutative**. The logical OR & logical AND operations of two Boolean variables x & y are shown below

$$x + y = y + x$$

$$x.y = y.x$$

Distributive Law

If any logical operation can be distributed to all the terms present in the Boolean function, then that logical operation is said to be **Distributive**. The distribution of logical OR & logical AND operations of three Boolean variables x, y & z are shown below.

$$x.\ y+z\ = x.y + x.z$$

$$x +\ y.z\ =\ x+y\ .\ x+z$$

Associative Law

If a logical operation of any two Boolean variables is performed first and then the same operation is performed with the remaining variable gives the same result, then that logical operation is said to be **Associative**. The logical OR & logical AND operations of three Boolean variables x, y & z are shown below.

$$x +\ y+z\ =\ x+y\ + z$$

$$x.\ y.z\ =\ x.y\ .z$$

Theorems of Boolean Algebra

The following two theorems are used in Boolean algebra.

- Duality theorem
- DeMorgan's theorem

Duality Theorem

This theorem states that the **dual** of the Boolean function is obtained by interchanging the logical AND operator with logical OR operator and zeros with ones. For every Boolean function, there will be a corresponding Dual function.

Let us make the Boolean equations *relations*

that we discussed in the section of Boolean postulates and basic laws into two groups. The following table shows these two groups.

Group1	Group2
$x + 0 = x$	$x.1 = x$
$x + 1 = 1$	$x.0 = 0$
$x + x = x$	$x.x = x$
$x + x' = 1$	$x.x' = 0$
$x + y = y + x$	$x.y = y.x$
$x + \quad y + z \quad = \quad x + y \quad + z$	$x. \quad y.z \quad = \quad x.y \quad .z$
$x. \quad y + z \quad = x.y + x.z$	$x + \quad y.z \quad = \quad x + y \quad . \quad x + z$

DeMorgan’s Theorem

This theorem is useful in finding the **complement of Boolean function**. It states that the complement of logical OR of at least two Boolean variables is equal to the logical AND of each complemented variable.

DeMorgan’s theorem with 2 Boolean variables x and y can be represented as

$$x + y\ ' = x'.y'$$

The dual of the above Boolean function is

$$x.y\ ' = x' + y'$$

Example 1

Let us **simplify** the Boolean function, f = p’qr + pq’r + pqr’ + pqr

We can simplify this function in two methods.

Method 1

Given Boolean function, f = p’qr + pq’r + pqr’ +pqr.

Step 1 − In first and second terms r is common and in third and fourth terms pq is common. So, take the common terms by using **Distributive law**.

$$\Rightarrow f = \quad p'q + pq' \quad r + pq \quad r' + r$$

Step 2 − The terms present in first parenthesis can be simplified to Ex-OR operation. The terms present in second parenthesis can be simplified to ‘1’ using **Boolean postulate**

$$\Rightarrow f = \quad p \oplus q \quad r + pq \quad 1$$

Step 3 − The first term can’t be simplified further. But, the second term can be simplified to pq using **Boolean postulate**.

$$\Rightarrow f = \quad p \oplus q \quad r + pq$$

Method 2

Given Boolean function, f = p’qr + pq’r + pqr’ + pqr.

Step 1 − Use the **Boolean postulate**, x + x = x. That means, the Logical OR operation with any Boolean variable ‘n’ times will be equal to the same variable. So, we can write the last term pqr two more times.

$$\Rightarrow f = p'qr + pq'r + pqr' + pqr + pqr + pqr$$

Step 2 − Use **Distributive law** for 1st and 4th terms, 2nd and 5th terms, 3rd and 6th terms.

$$\Rightarrow f = qr\ p' + p\ \ + pr\ q' + q\ \ + pq\ r' + r$$

Step 3 − Use **Boolean postulate**, x + x’ = 1 for simplifying the terms present in each parenthesis.

$$\Rightarrow f = qr\ 1\ \ + pr\ 1\ \ + pq\ 1$$

Step 4 − Use **Boolean postulate**, x.1 = x for simplifying the above three terms.

$$\Rightarrow f = qr + pr + pq$$

$$\Rightarrow f = pq + qr + pr$$

Any Query????

Thank you……