SNS COLLEGE OF ENGINEERING

Coimbatore-107

COURSE NAME: ANALYSIS OF ALGORITHM

II YEAR/ IV SEMESTER

UNIT – III

DYNAMIC PROGRAMMING

Topic

Dynamic Programming: Binomial Coefficient

Unit III - Dynamic Programming

Dynamic programming is a method in which the solution to a problem is obtained by making sequence of decisions.

* Optimal solution is obtained from sequence of all possible solutions generated.
* It is given by U.S. Mathematician Richard Bellman in 1950.
* It solves problems with overlapping sub problems.

(i). Computing Binomial coefficient:

In combinatorics, Binomial coefficient is a coefficient of any of the terms in the expansion of $(a+b)^n$.

* It is denoted by $C(n,k)$ or $\binom{n}{k}$ where $(0 \le k \le n)$

Formula for Computing Binomial Coefficient

Pascal Identity

$$C(n,k) = C(n-1, k-1) + C(n-1, k)$$

and $C(n,0) = 1$, $C(n,n) = 1$; $n > k > 0$

$$(a+b)^n = \sum_{k=0}^{n} C(n,k)\, a^{n-k} b^k$$

classmate

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Example: Compute $C(4,2)$

$\boxed{n=4,\ k=2;}$

(i) Compute $C(4,2) = C(n-1, k-1) + C(n-1), k)$

$C(4,2) = C(4-1, 2-1) + C(4-1), 2)$

Eq ① — $C(4,2) = C(3,1) + C(3,2)$

Both $C(3,1)$ & $C(3,2)$ are unknown.
So compute it, to solve Equn. 1.

$\boxed{n=3,\ k=1}$

(ii) Compute $C(3,1) = C(n-1, k-1) + C(n-1), k$

$= C(3-1, 1-1) + C(3-1), 1$

$= C(2,0) + C(2,1)$

(iii) Compute $C(3,2) = C(n-1, k-1) + C(n-1, k)$

$= C(3-1, 2-1) + C(3-1, 2)$

$= C(2,1) + C(2,2)$

We should solve $C(2,0)$, $C(2,1)$, $C(2,2)$ to solve equn ①

(iii) Compute $C(2,0)$ and $C(2,2)$

From formula. $C(n,0) = 1$;

$\therefore C(2,0) = 1$.

$C(2,2)$

$C(n,n) = 1$

$\boxed{\therefore C(2,2) = 1}$

$n=2$; $k=1$

(iv) Compute $C(2,1) = C(n-1, k-1) + C(n-1, k)$

$= C(2-1, 1-1) + C(2-1, 1)$

$= C(1,0) + C(1,1)$

By formula, we know $c(n,0)=1$ & $c(n,n)=1$.

$$\therefore c(2,1) = c(1,0) + c(1,1)$$
$$= 1 + 1$$
$$\boxed{\therefore c(2,1) = 2}$$

(v) Apply values $c(2,1)$, $c(2,0)$ & $c(2,2)$ in $c(3,1)$ & $c(3,2)$ we get,

$$c(3,1) = c(2,0) + c(2,1)$$
$$= 1 + 2$$
$$\boxed{\therefore c(3,1) = 3}$$

$$c(3,2) = c(2,1) + c(2,2)$$
$$= 2 + 1$$
$$\boxed{\therefore c(3,2) = 3}$$

(vi) Apply the values of $c(3,1)$ and $c(3,2)$ in Equn. (1)

$$c(4,2) = c(3,1) + c(3,2)$$
$$= 3 + 3$$
$$\boxed{\therefore c(4,2) = 6}$$

To compute $c(n,k)$, smaller overlapping sequences generated by
=) $c(n-1, k-1)$ & $c(n-1, k)$.
=) Smaller instances solved first.
=) These solutions generate final solution.

Algorithm:

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Algorithm Binomial (n,k)
for (i=0 to n) do
for (j=0 to k) do
if (j=0 (or) (i==j)) then
c[i,j]=1
else
c[i,j] = c[i-1,j-1] + c[i-1,j]
}
return c[n,k].
```

Analysis: Time Complexity

$T(n) = O(nk)$

since we fill $(n+1) * (k+1)$ table

Space Complexity:

$S(n) = O(nk)$.