SNS COLLEGE OF ENGINEERING

Coimbatore-107

COURSE NAME: ANALYSIS OF ALGORITHM

II YEAR/ IV SEMESTER

UNIT – II

BRUTE FORCE METHOD

Topic

Convex-Hull Problems

Unit II. Convex Hull Problem

The convex Hull of set of points is the smallest convex shape that encloses all the points.

* If checks every pair of points to see if it forms edge of convex Hull.
* If is also called Jarvis march (or) Gift wrapping Algorithm.

Example:

Note: => All edge must have all other points on one side (All above or All Below the line).

=> If a pair satisfies the condition, it is part of Convex Hull.

Example: Step Next:

Use determinant Formula:

For 3 points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ Compute:

$$\text{Value} = (x_2 - x_1) \cdot (y_3 - y_1) - (y_2 - y_1) \cdot (x_3 - x_1)$$

(x_1, y_1) & (x_2, y_2) are 2 points forming the line.

x_3, y_3 → point we check.

If value > 0, P3 is on one side of line

If value < 0, P3 is on other side

If value = 0; P3 is on the line.

Example:

(0,0), (2,2), (2,0), (1,3)

Step 1:

Consider all line segments between two points.

1. (0,0) → (2,2)
2. (0,0) → (2,0)
3. (0,0) → (1,3)
4. (2,2) → (2,0)
5. (2,2) → (1,3)
6. (2,0) → (1,3)

Now check whether all other points are on the same side, for each line segment.

Step 2:

Use cross product formula

$$(x_2 - x_1) \cdot (y - y_1) - (y_2 - y_1) \cdot (x - x_1)$$

Result = +ve (or) -ve; all are on same side & Segment belong to convex hull

Result = +ve (and) -ve ; ⇒ Points on different sides
⇒ Segment is not part of convex Hull.

Step 3:

I. Now check each line segment.

(i) Equation : Point (x, y) $(0,0)$ & $(2,2)$

Remaining points : $(2,0)$ & $(1,3)$

Checking point : $(2,0)$ ✓

$x_1 = 0$, $y_1 = 0$, $x_2 = 2$, $y_2 = 2$.

$x = 2$, $y = 0$

Apply in formula & weget ;

$$(2-0)\cdot(0-0) - (2-0)\cdot(2-0) = 0 - 4 = -4$$

We get Negative and positive values, points are on different side.

Value = -4 & 4

∴ $(0,0)$ – $(2,2)$ is not part of convex Hull.

(ii). Now check line $(0,0)$ & $(2,2)$

Equation : Points (x_1, y_1) $(0,0)$ & (x_2, y_2) $(2,2)$

Checking point : $(1,3)$ ✓

$x_1 = 0$, $y_1 = 0$, $x_2 = 2$, $y_2 = 2$; $x = 1$, $y = 3$

Apply in formula, we get,

~~$(2-0)\cdot(3-0) - (2-0)\cdot(2-0) = 0 =$~~

$$(2-0)\cdot(3-0) - (2-0)\cdot(1-0) = 6 - 2 = 4$$

II. Now checking Line Segment (0,0) → (2,2)

Remaining Points: (2,2), (1,3)

(i). checking point : (2,2) : x_1, y_1 x_2, y_2

$x_1 = 0, y_1 = 0, x_2 = 2, y_2 = 0, x = 2, y = 2$

Apply in formula, we get

$(2-0)\cdot(2-0) - (0-0)\cdot(2-0) = 4 - 0 = 4$

(ii) checking point : (1,3)

$x_1 = 0, y_1 = 0, x_2 = 2, y_2 = 0, x = 1, y = 3$

Apply in formula, we get

$(2-0)\cdot(3-0) - (0-0)\cdot(1-0) = 6 - 0 = 6$

Values = 4 & 6

Both Values are positive, all point are on same side.

∴ (0,0) – (2,0) is part of convex

III. checking Line Segment : (0,0) → (1,2) : x_1, y_1 x_2

(i) checking point : (2,2) $x = 2, y = 2$

Remaining Point: (2,2) (1,0)

Apply in formula, we get

$(1-0)\cdot(2-0) - (3-0)\cdot(2-0) = 2 - 6 = -4$

(1 = 0).

(ii) checking point : (2,0) $x = 2, y = 0$

$(1-0)\cdot(0-0) - (3-0)\cdot(2-0) = 0 - 6 = -6$

Values = −4 & −6

Both values are negative; all points on the same side.

∴ (0,0) → (1,3) is part of convex Hull.

IV. checking line $(x_1, y_1) = (2,2) \rightarrow (x_2, y_2) = (2,0)$

Remaining points: (0,0), (1,3)

(i) checking point: (0,0) $x=0, y=0$

$x_1=2, x_2=2, y_1=2, y_2=0, x=0, y=0$

Apply in formula, we get,

$(2-2)\cdot(0-2) - (0-2)\cdot(0-2) = 0-4 = -4$

(ii) checking point: (1,3) $x=1, y=3$

$(2-2)\cdot(3-2) - (0-2)\cdot(1-2) = 0-2 = -2$

Both values are negative,

All points are on the same side.

Values = −4 & −2

∴ (2-2) → (2,0) is a part of convex Hull.

V. checking line $(x_1, y_1) = (2,2) \rightarrow (x_2, y_2) = (1,3)$

Remaining points: (0,0), (2,0)

(i) checking point: (0,0) $x=0, y=0$

$x_1=2, x_2=2, y_1=1, y_2=3, x=0, y=0$

Apply in formula, we get,

$(1-2)\cdot(0-2) - (3-2)\cdot(0-2) = 2-2 = 0$

(ii) checking point : $(2,0)$ $x\ y$ Values = 0 & 2

$(1-2)\cdot(0-2) - (3-2)\cdot(2-2) = 2-0 = 2$

We get 0 and positive values points are not on one side $(2-2) \rightarrow (1,3)$ is not a past of convex Hull.

vi. checking line $(2,0) \rightarrow (1,3)$ $x_1\ y_1$ $x_2\ y_2$

Remaining points : $(0,0)$ $(2,2)$ $x\ y$ Checkpoint = (0,

(i) $x_1 = 2$, $y_1 = 0$, $x_2 = 1$, $y_2 = 3$, $x = 0$, $y =$

Apply in formula, we get

$(1-2)\cdot(0-0) - (3-0)\cdot(0-2) = 0+6 = 6$

(ii) checking point $(2,2)$ $x\ y$

Apply in formula, we get

$(1-2)\cdot(2-0) - (3-0)\cdot(2-2) = -2-0 =$

We get postive & negatives & so points are on different side.

$(2,0) \rightarrow (1,3)$ is not a past of convex Hull.

Values = 6 & -2

Line Segment	other points	sign of cross product	part of convex Hull
1. (0,0) → (2,2)	(2,0), (1,3)	mixed (+ and −)	NO
2. (0,0) → (2,0)	(2,2), (1,3)	same (+ and +)	yes
3. (0,0) → (1,3)	(2,2), (2,0)	same (− and −)	yes
4. (2,2) → (2,0)	(0,0), (1,3)	same (− and −)	yes
5. (2,2) → (1,3)	(0,0), (2,0)	mixed (+ and −)	NO
6. (2,0) → (1,3)	(0,0), (2,2)	mixed (+ and −)	NO

The points forming convex Hull are ↳ polygons.

// (0,0), (2,0), (2,2), (1,3) //

Algorithm:

```
Algorithm convexHull(points[][2], n)
{
  for(i=0; i<n; i++)
  {
  for(j=i+1; j<n; j++)
   {
    for (k=0; k<n; k++)
    {
      if (k==i || k==j)
```

```
cp = crossprod(points[i][0], points[i][1],
     Points[j][0], points[j][1], Points[k][0],
     Points[k][1]);
if(cp > 0) pos++;
else (cp < 0) neg++;
}
// if(pos == 0 || neg == 0)
// Add edge;
}
}
}
}
```

Algorithm crossprod(x1, y1, x2, y2, x, y)

```
{
return (x2 - x1) * (y - y1) - (y2 - y1) * (x - x1
}
```

Time Complexity:

$O(n^3)$ → { checking $O(n^2)$ pairs & verifying $O(n)$ points for each pair

Space complexity:

$O(1)$ → use only few extra variables.