TOPIC: 2.2.INTERPOLATION WITH UNEQUAL INTERVALS - LAGRANGE'S INTERPOLATION

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UNIT-II INTERPOLATION AND APPROXIMATION:

Lagrangian polynomials – Divided differences – Interpolating with a cubic Spline – Newton's forward and backward difference formulas.

Lagrangian polynomials:

Lagrangian polynomial is a simplest straight forward approach for interpolating an evenly space data.

The Lagrangian polynomial interpolation formula,

$$y=f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y_1 + \cdots + \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}y_n$$

Problems:

1. Using Lagrangian polynomial Interpolation calculated the Profit in the year 2000 from the following data.

Year (x) :	1997	1999	2001	2002
Profit Lakhs(y):	43	65	159	248

Solution:

The Lagrangian polynomial Interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n$$

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3.$$

$$= \frac{(x-1999)(x-2001)(x-2002)}{(1997-1999)(1997-2001)(1997-2002)}(43) + \frac{(x-1997)(x-2001)(x-2002)}{(1999-1997)(1999-2001)(1999-2002)}(65)$$

$$+ \frac{(x-1997)(x-1999)(x-2002)}{(2001-1997)(2001-1999)(2001-2002)}(159) + \frac{(x-1997)(x-1999)(x-2001)}{(2002-1997)(2002-1999)(2002-2001)}(248)$$

$$y(2000) = \frac{(2000-1999)(2000-2001)(2000-2002)}{2(-4)(-5)}(43) + \frac{(2000-1997)(2000-2001)(2000-2002)}{(2)(-2)(-3)}(65)$$

$$+ \frac{(2000-1997)(2000-1999)(2000-2002)}{(4)(2)(-1)}(159) + \frac{(2000-1997)(2000-1999)(2000-2001)}{(5)(3)(1)}(248)$$

$$= \frac{(1)(-1)(-2)}{2(-4)(-5)} + \frac{(3)(-1)(-2)}{(2)(-2)(-3)}(65) + \frac{(3)(1)(-2)}{(4)(2)(-1)}(159) + \frac{(3)(1)(-1)}{(5)(3)(1)}(248)$$

$y = 100$

2. Using Lagrange's formula (2) fit the polynomial to the data.

x:	-1	1	2
y:	7	5	15

Solution:

The Lagrangian polynomial interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-1)(x-2)}{(-1-1)(-1-2)}(7) + \frac{(x+1)(x-2)}{(1+1)(1-2)}(5) + \frac{(x+1)(x-1)}{(2+1)(2-1)}(15)$$

$$= \frac{x^2-2x-x+2}{(-2)(-3)}(7) + \frac{(x^2-2x+x-2)}{(2)(-1)}(5) + \frac{(x^2-x+x-1)}{(3)(1)}(15)$$

$$= \frac{x^2-3x+2}{6}(7) + \frac{x^2-x-2}{-2}(5) + \frac{x^2-1}{3}(15)$$

$$= \frac{7x^2-21x+14}{6} - \frac{5x^2+5x+10}{2} + \frac{15x^2-15}{3}$$

$$= \frac{1}{6}\left[7x^2-21x+14-15x^2+15x+30+30x^2-30\right]$$

$$= \frac{1}{6}\left[22x^2-6x+14\right] = \frac{2}{6}\left[11x^2-3x+7\right] = \frac{1}{3}\left[11x^2-3x+7\right]$$

$$y = \frac{1}{3}\left[11x^2-3x+7\right]$$

3. Using Lagrange's formula fit a polynomial to the data

x: 0 1 3 4

y: -12 0 6 12 Also find y at $x=2$.

Solution:

The Lagrangian polynomial interpolation formula is

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}(0) + \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}(6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}(12)$$

$$= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)}(-12) + \frac{x(x-3)(x-4)}{(1)(-2)(-3)}(0) + \frac{(x)(x-1)(x-4)}{3(2)(-1)}(6) + \frac{x(x-1)(x-3)}{(4)(3)(1)}(12)$$

$$y(2) = \frac{(2-1)(2-3)(2-4)}{(-1)(-3)(-4)}(-12) + \frac{2(2-3)(2-4)}{(1)(-2)(-3)}(0) + \frac{(2)(2-1)(2-4)}{-6}(6) + \frac{(2)(2-1)(2-3)}{12}(12)$$

$$= \frac{(1)(-1)(-2)}{-12}(-12) + \frac{(2)(1)(-2)}{-6}(6) + \frac{(2)(1)(-1)}{12}(12)$$

$$= 2+4-2 = 4.$$

$$y(2) = 4.$$