



## TOPIC : 2.3 – LAGRANGE'S INVERSE INTERPOLATION

2. Using Lagrange's formula fit the polynomial to the data.

x:	-1	1	2
y:	7	5	15

Solution:

The Lagrangian polynomial interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-1)(x-2)}{(-1-1)(-1-2)} (7) + \frac{(x+1)(x-2)}{(1+1)(1-2)} (5) + \frac{(x+1)(x-1)}{(2+1)(2-1)} (15)$$

$$= \frac{x^2 - 2x - x + 2}{(-2)(-3)} (7) + \frac{(x^2 - 2x + x - 2)}{(2)(-1)} (5) + \frac{(x^2 - x + x - 1)}{(3)(1)} (15)$$

$$= \frac{x^2 - 3x + 2}{6} (7) + \frac{x^2 - x - 2}{-2} (5) + \frac{x^2 - 1}{3} (15)$$

$$= \frac{7x^2 - 21x + 14}{6} - \frac{5x^2 + 5x + 10}{2} + \frac{15x^2 - 15}{3}$$

$$= \frac{1}{6} [7x^2 - 21x + 14 - 15x^2 + 15x + 30 + 30x^2 - 30]$$

$$= \frac{1}{6} [22x^2 - 6x + 14] = \frac{2}{6} [11x^2 - 3x + 7] = \frac{1}{3} [11x^2 - 3x + 7]$$

$$y = \frac{1}{3} [11x^2 - 3x + 7]$$



3. Using Lagrange's formula fit a polynomial to the data

$x_i$	0	1	3	4
$y_i$	-12	0	6	12

Also find  $y$  at  $x=2$ .

Solution:

The Lagrangean polynomial interpolation formula is

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$\begin{aligned} & \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 \\ &= \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)} (-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)} (0) + \\ & \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} (6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)} (12) \\ &= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)} (-12) + \frac{x(x-3)(x-4)}{(1)(-2)(-3)} (0) + \\ & \frac{(x)(x-1)(x-4)}{3(2)(-1)} (6) + \frac{x(x-1)(x-3)}{(4)(3)(1)} (12) \\ y(2) &= \frac{(2-1)(2-3)(2-4)}{(-1)(-3)(-4)} (-12) + \frac{2(2-3)(2-4)}{(1)(-2)(-3)} (0) + \\ & \frac{(2)(2-1)(2-4)}{-6} (6) + \frac{(2)(2-1)(2-3)}{12} (12) \\ &= \frac{(1)(-1)(-2)}{-12} (-12) + \frac{(2)(1)(-2)}{-6} (6) + \frac{(2)(1)(-1)}{12} (12) \\ &= 2 + 4 - 2 = 4. \\ y(2) &= 4. \end{aligned}$$