TOPIC : 2.5.NEWTON'S DIVIDED DIFFERENCE INTERPOLATION

DIVIDED DIFFERENCES :

Divided difference method avoids all in comparision and arithematic operation are used in Lagrangian polynomial method.

Newton's divided difference formula is

$$y(x) = y_0 + (x-x_0)\,f(x_0,x_1) + (x-x_0)(x-x_1)\,f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)\,f(x_0,x_1,x_2,x_3) + \cdots\cdots$$

NEWTON'S DIFFERENCE TABLE: (3)

Arguments x	Entry $y=f(x)$	1st D.D $\Delta f(x)$	2nd D.D $\Delta^2 f(x)$	3rd D.D $\Delta^3 f(x)$	4th D.D $\Delta^4 f(x)$
x_0	y_0				
		$\frac{y_1-y_0}{x_1-x_0}=k_1$			
x_1	y_1		$\frac{k_2-k_1}{x_2-x_0}=k_{11}$		
		$\frac{y_2-y_1}{x_2-x_1}=k_2$		$\frac{k_{12}-k_{11}}{x_3-x_0}=k_{111}$	
x_2	y_2		$\frac{k_3-k_2}{x_3-x_1}=k_{12}$		$\frac{k_{112}-k_{111}}{x_4-x_0}$
		$\frac{y_3-y_2}{x_3-x_2}=k_3$		$\frac{k_{13}-k_{12}}{x_4-x_1}=k_{112}$	
x_3	y_3		$\frac{k_4-k_3}{x_4-x_2}=k_{13}$		
		$\frac{y_4-y_3}{x_4-x_3}=k_4$			
x_4	y_4				

Problems:

1. Find the cubic function from the following table

x :	0	1	3	4
$f(x)$:	1	4	40	85.

Solution:

The Newton's Divided Difference table is

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1	$\frac{4-1}{1-0}=3$	$\frac{18-3}{3-0}=5$	$\frac{9-5}{4-0}=1$
1	4	$\frac{40-4}{3-1}=18$	$\frac{45-18}{4-1}=9$	
3	40	$\frac{85-40}{4-3}=45$		
4	85			

Newton's divided difference formula is,

$$y(x) = y_0 + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3).$$

$$= 1 + (x)(3) + (x)(x-1)(5) + (x)(x-1)(x-3)(1)$$

$$= 1 + 3x + [x^2 - x(5)] + [x^3 - 3x^2 - x^2 + 3x]$$

$$= 1 + 3x + 5x^2 - 5x + x^3 - 3x^2 - x^2 + 3x.$$

$$f(x) = x^3 + x^2 + x + 1.$$

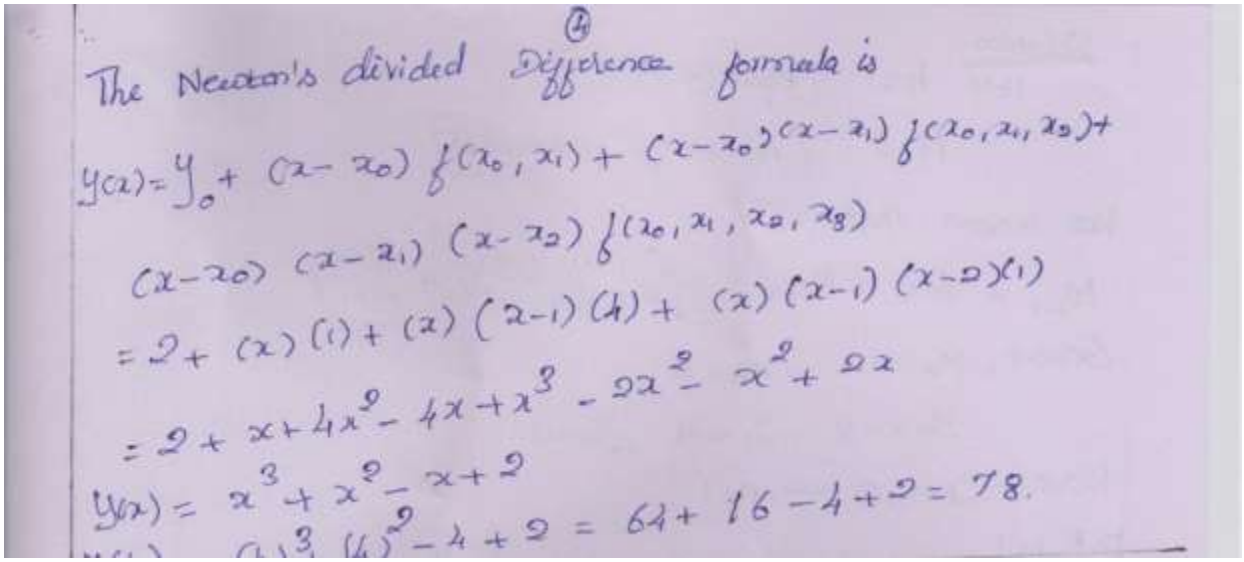
2. Find the cubic polynomial from the following table and hence find $y(4)$:

x:	0	1	2	5
$y = f(x)$:	2	3	12	147

Solution:

The Newton's divided difference formula is

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2	$\frac{3-2}{1-0} = 1$	$\frac{9-1}{2-0} = 4$	$\frac{9-4}{5-0} = 1$
1	3	$\frac{12-3}{2-1} = 9$	$\frac{45-9}{5-1} = 9$	
2	12	$\frac{147-12}{5-2} = 45$		
5	147			



The Newton's divided Difference formula is

$$y(x) = y_0 + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$= 2 + (x)(1) + (x)(x-1)(4) + (x)(x-1)(x-2)(1)$$

$$= 2 + x + 4x^2 - 4x + x^3 - 2x^2 - x^2 + 2x$$

$$y(x) = x^3 + x^2 - x + 2$$

$$y(4) = (4)^3 + (4)^2 - 4 + 2 = 64 + 16 - 4 + 2 = 78.$$