TOPIC: 2.7.NEWTON'S FORWARD INTERPOLATION FORMULA

Newton's forward and backward difference formulas:-

Newton's forward formula: [For equal intervals] (Δ):

Let $y=f(x)$ denote a function which takes the values $y_0, y_1, y_2, \dots, y_n$ corresponding to the values $x_0, x_1, \dots, x_n$ respectively of x. The value of 'x' are equi-distant.

Note:

The formula is used to interpolate the values of y nearer to the beginning value of the table.

Newton's backward Interpolation formula:

[For equal interval] (∇):

Let $y=f(x)$ denote a function which takes the values $y_0, y_1, \dots, y_n$ corresponding to the values $x_0, x_1, \dots, x_n$ respectively of x. The value of 'x' are equi-distant.

$$y(x) = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \cdots$$

where $n = \frac{x - x_n}{h}$

The formula is used to interpolate the values of y nearer to the end table.

Problems:

1. Find the values of y at $x=21$ and $x=28$ from the following data

x:	20	23	26	29
$y(x)$:	0.3420	0.3907	0.4384	0.4848

Solution.

Difference table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
20	0.3420			
		0.0487		
			−0.001	
				−0.0005
23	0.3907			
		0.0477		
			−0.0015	
26	0.4384			
		0.02164		
29	0.4848			

Newton's forward formula is

$$y(x) = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

Here $n = \frac{x - x_0}{h}$, $x = 21$.

$$= \frac{21-20}{3} = \frac{1}{3} = 0.333$$

$$y(21) = 0.3420 + \frac{0.333}{1!}(0.0487) + \frac{(0.333)(0.3337)}{2!}(-0.001)$$

$$+ \frac{(0.333)(-0.6667)(-1.6667)}{3!}(-0.0005)$$

$y(21) = 0.3583$

ii) Newton's backward difference formula is

$$n = \frac{x - x_n}{h} = \frac{28-29}{3} \quad (x = 28)$$

$$= -\frac{1}{3} = -0.3333$$

$$y(28) = 0.4848 + \frac{(-0.3333)}{1}(0.0464) + \frac{(-0.3333)}{2}(-0.0015) + \frac{(-0.3333)(0.667)(1.667)}{6}(-0.00\ldots$$