F - Test :-

To test whether if there is any significant difference between two population variance.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$.

LOS : 1% or 5%

Dof : $V_1 = n_1 - 1$ $V_2 = n_2 - 1$.

Test Statistics

$$F = \frac{S_1^2}{S_2^2} = \frac{\text{Greater variance}}{\text{Smaller variance}}$$

$$= S_2^2 / S_1^2$$

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$$

$$S_1^2 > S_2^2 \quad (\text{or}) \quad S_2^2 > S_1^2$$

Problems:-

1. In one sample of 10 observations from a normal population, the sum of the squares of the deviations of the sample values from the sample mean is 102.4 and in another sample of 12 observations from another normal population, the sum of the squares the the deviations of the sample values from the

Sample mean is 120.5. Examine whether the two normal Population have the same variance.

$n_1 = 10$ $\quad n_2 = 12$

$\Sigma(x_1 - \bar{x}_1)^2 = 102.4$ $\quad \Sigma(x_2 - \bar{x}_2)^2 = 120.5$

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$= \frac{102.4}{9}$$

$$= 11.37$$

$$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$$

$$= \frac{120.5}{11}$$

$$10.95.$$

$S_1^2 > S_2^2$.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$.

LOS = 5 %

Dof $v_1 = n_1 - 1 = 9$ $v_2 = n_2 - 1 = 11$.

Test statistics.

$$F = \frac{S_1^2}{S_2^2} = \frac{11.37}{10.95} = 1.038.$$

Critical value.

$\alpha = 5\%$ $v_1 = 9$ $v_2 = 11$.

$F_\alpha = 2.90$.

Conclusion: C.V T.V.

$1.132 < 2.90$

H_0 accepted.

2). Two random samples gave the following results.

Sample	Size	Sample mean	Sum of squares of deviation from mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population.

$n_1 = 10$
$\bar{x}_1 = 15$
$\sum (x_1 - \bar{x}_1)^2 = 90$

$n_2 = 12$
$\bar{x}_2 = 14$
$(x_2 - \bar{x})^2 = 108$

$S_1^2 = \frac{90}{9} = 10$ $S_2^2 = \frac{108}{11} = 9.82$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

LOS: 5%.

Dof: $V_1 = 10-1 = 9$ $V_2 = 12-1 = 11$.

Test Statistics

$$F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.018$$

Critical value: $\alpha = 5\%$ at $(9, 11)$

Critical value: $\alpha = 5\%$ at $(9, 11)$

$F_\alpha = 2.90$.

Conclusion: C.V T.V

$1.018 < 2.90$

H_0 accepted.