TOPIC : 3.5. t - distribution for mean

Tests for small sample :

1. Student's 't' test or 't' test
2. F-test
3. χ^2 - test

Student's 't' Test for single mean :

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$

Where $\bar{x}$ - sample mean

μ - population mean

n - sample size

s - standard deviation of sample.

Conclusion :

Calculated value   Tabulated value.

$|t| < |t_\alpha| \Rightarrow H_0$ accepted

$|t| > |t_\alpha| \Rightarrow H_0$ rejected.

Note:

If Standard deviation is not given directly, (ie). if Samples are given then $(x_1, x_2, \ldots)$

$$\text{Mean } \bar{x} = \frac{\Sigma x}{n}$$

$$\text{Variance } s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1}$$

$$\text{test statistic } t = \frac{\bar{x} - \mu}{\frac{s^2}{\sqrt{n}}}$$

1. The mean life time of a sample of 25 fluorescent light bulbs produced by a company is computed to be 1570 hours with a S.D of 120 hrs. The company claims that the average life of the bulbs produced by the company is 1600 hrs. Using the level of significance of 0.05. Is the claim acceptable?

Soln: Given Sample size, $n = 25$

Sample mean $\bar{x} = 1570$

Population mean $\mu = 1600$

S.D $(s) = 120$.

Degrees of freedom $= n-1 = 25-1 = 24$

Null hypothesis: (H_0)

The claim is acceptable, $\mu = 1600$ hrs

Alternative hypothesis: (H_1)

$\mu \neq 1600$ hrs.

Test statistic $t = \dfrac{\bar{x} - \mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)}$

$= \dfrac{1570 - 1600}{\left(\dfrac{120}{24}\right)} = \dfrac{-30}{24.49}$

$t = -1.22$

$|t| = 1.22$

Calculated value $t = 1.22$

Tabulated value:

Level of significance $= 0.05$ or 5%.

Degrees of freedom $= 24$

for two tailed test $\Rightarrow t_\alpha = 2.06$.

$|t| < |t_\alpha|$

calculated value $<$ tabulated value.

$\therefore$ We accept the null hypothesis H_0. (ie) the claim that the average life of the bulbs produced by the company is 1600 hrs is acceptable.

2. The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a S.D of 17.2. Was the advertising campaign successful.

Soln

Given, number of stores $n = 22 < 30$.

$\therefore$ The sample is small.

Sample mean $\bar{x} = 153.7$

Population mean $\mu = 146.3$

S.D, $S = 17.2$

Degrees of freedom $= n-1 = 21$.

Null hypothesis: H_0

The advertising campaign was not successful

Alternative hypothesis: H_1

$H_1 > 146.3$ (Right tail)

Test statistic $t = \dfrac{\bar{x} - \mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)}$

$$= \frac{153.7 - 146.3}{\dfrac{17.2}{\sqrt{22-1}}} = 1.97$$

calculated value $t = 1.97$

Tabulated value:

L.O.S $= 5\%$.

Degrees of freedom $= 21$.

for single tailed test $\Rightarrow t_\alpha = 1.72$

tabulated value, $t_\alpha = 1.72$

Conclusion: $|t| > |t_\alpha|$ $(1.97 > 1.72)$

calculated value > tabulated value

$\therefore$ We reject null hypothesis H_0.

$\therefore$ Advertising campaign was successful.

3. A random sample of 10 boys had the following IQ's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q of 100?

Soln. Here S.D and mean of the sample is not given directly, we have to determine these S.D and mean as follows.

x	$x-\bar{x}$	$(x-\bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
$\Sigma x = 972$		$\Sigma(x-\bar{x})^2 = 1833.60$

Mean $\bar{x} = \frac{\Sigma x}{n} = \frac{972}{10} = 97.2.$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$$

$$= \frac{1833.60}{9} = 203.73$$

$$S.D, s = \sqrt{s^2} = \sqrt{203.73} = 14.27$$

Null hypothesis: H_0

The data support the assumption of a population mean I.Q. of 100 in the population

(ie) $\mu = 100$.

Alternative Hypothesis: H_1

$$\mu \neq 100.$$

Test Statistic:

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{97.2 - 100}{\left(\frac{14.27}{\sqrt{10}}\right)}$$

$$= \frac{-2.8}{4.5/\sqrt{10}} = -0.62.$$

$|t| = 0.62. \Rightarrow$ calculated value $= 0.62$

Tabulated value :

degrees of freedom $= n-1 = 10-1 = 9$

L.O.S $= 5\%$.

for two tailed test $\Rightarrow t_\alpha = 2.26$.

Conclusion :

$0.62 < 2.26$

Calculated value < tabulated value.

$\therefore$ We accept the null hypothesis H_0.

$\therefore$ The data support the assumption of mean I.Q of 100 in the population.