TOPIC : 3.2 Large sample test based on Normal distribution for single mean

Large Sample :

If the size of the sample $n > 30$, then that sample is called large sample.

Z-Test

Test of significance for single mean :

Null hypothesis $H_0 : \mu = \mu_0$

Alternative hypothesis $H_1 : \mu \neq \mu_0$

Test statistic : [calculated value, Z]

$$|Z| = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where σ is the S.D of the population.

If the population S.D is not known then

$$|Z| = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

where s is the sample S.D

Level of Significance :

5 % or 1 %

Tabulated value Z_α :

	LOS	
	1%	5%
Two tailed test	2.58	1.96
one tailed test	2.33	1.645

Conclusion :

If calculated value $Z <$ Tabulated value Z_α

(ie) $|Z| < Z_\alpha$

$\Rightarrow$ we accept the null hypothesis H_0

If $|Z| > Z_\alpha$

$\Rightarrow$ we reject the null hypothesis H_0.

1. A sample of 900 members has a mean of 3.4 cms and S.D 2.61 cms. Is the sample from a large population of mean 3.25 cm and S.D 2.61 cms.

Soln.

Null hypothesis H_0:

Assume that the sample has been drawn from the population with mean $\mu = 3.25$

Alternative hypothesis H_1:

$\mu \neq 3.25$

Test statistic:

$$|Z| = \left| \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \right|$$

Given $n = 900$

$\mu = 3.25$ cm $\quad \bar{x} = 3.4$ cm

$\sigma = 2.61$ cm $\quad s = 2.61$ cm

$$|Z| = \left| \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}} \right|$$

$$= 1.724$$

Level of Significance: [LOS]

LOS = 5 % (Two tailed test)

$Z_\alpha = 1.96$

Conclusion:

$1.724 < 1.96$

$|Z| < Z_\alpha$

We accept the null hypothesis H_0.

. The sample has been drawn from the population with mean $\mu = 3.25$

2. A random sample of 200 employee's at a large corporation showed their average age to be 42.8 yrs with a S.D of 6.8 yrs Test the hypothesis $H_0 : \mu = 40$ against $H_1 : \mu > 40$ at $\alpha = 0.01$ LOS

Sol:- Null hypothesis $H_0 : \mu = 40$

Alternative hypothesis $H_1 : \mu > 40$

(one tailed Right).

$\mu = 40$, $n = 200$ $\bar{x} = 42.8$ yrs

$\sigma = 6.8$ yrs

Test Statistic:

$$|Z| = \left| \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \right| = \left| \frac{42.8 - 40}{6.8/\sqrt{200}} \right| = 5.747$$

L.O.S = 1 %.

Critical value (calculated value)

At 1%. (one tailed test)

$Z_\alpha = 2.33$

Conclusion:

$5.747 > 2.33$

$|Z| > Z_\alpha$

we reject the null hypothesis H_0.