🌟 Puzzle 1: Analytic or Not?

Consider the complex function:

$$f(z) = \mathrm{Re}(z^2) + i\,\mathrm{Im}(z^2)$$

where $z = x + iy$.

1. Is this function analytic?
2. If yes, find its derivative. If not, explain why using the Cauchy-Riemann equations.

🎯 Puzzle 2: Mysterious Mapping

A function $f(z) = \frac{1}{z}$ maps the complex plane $\mathbb{C}$ (except at $z = 0$).

1. What happens to the unit circle $|z| = 1$ under this mapping?
2. Describe geometrically what happens to lines in the z-plane under this transformation.

🔥 Puzzle 3: Contour Chaos

Evaluate the contour integral:

$$\int_C \frac{dz}{z^2 + 1}$$

where C is the unit circle $|z| = 1$, traversed counter-clockwise.

(Hint: Identify the singularities inside the contour and apply the residue theorem!)