TOPIC : GAUSS JORDAN METHODS

GAUSS - JORDAN METHOD:

Problems:

1. Solve $x+3y+3z=16$, $x+4y+3z=18$, $x+3y+4z=19$, by Gauss-Jordan method.

Solution:

The given system of equation can be written in matrix form $AX=B$.

$$\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 18 \\ 19 \end{pmatrix}$$

$\therefore$ The augmented matrix is

$$(A/B) \sim \begin{pmatrix} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 3 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} R_1 \to R_1 - 3R_2$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} R_1 \to R_1 - 3R_1$$

$\therefore x = 1$

$y = 2$

$z = 3$

3. Using the Gauss Jordan method solve the following equations $10x+y+z=12$, $x+10y+z=12$, $x+y+10z=12$

Solution:

The given system of equation can be written in matrix form $AX=B$

$$\begin{pmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ 12 \end{pmatrix}$$

$\therefore$ The augmented matrix is

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$$(A/B) \sim \begin{pmatrix} 10 & 1 & 1 & 12 \\ 1 & 10 & 1 & 12 \\ 1 & 1 & 10 & 12 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 10 & 1 & 12 \\ 10 & 1 & 1 & 12 \\ 1 & 1 & 10 & 12 \end{pmatrix} \quad R_1 \longleftrightarrow R_2$$

$$\sim \begin{pmatrix} 1 & 10 & 1 & 12 \\ 0 & -99 & -9 & -108 \\ 0 & -9 & 9 & 0 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 10R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 10 & 1 & 12 \\ 0 & -99 & -9 & -108 \\ 0 & 0 & 108 & 108 \end{pmatrix} \quad R_3 \to 11R_3 - R_2$$

$$\sim \begin{pmatrix} 99 & 0 & -9 & 108 \\ 0 & -99 & -9 & -108 \\ 0 & 0 & 108 & 108 \end{pmatrix} \quad R_1 \to 99R_1 + 10R_2$$

$$\sim \begin{pmatrix} 10692 & 0 & 0 & 10692 \\ 0 & -99 & -9 & -108 \\ 0 & 0 & 108 & 108 \end{pmatrix} \quad R_1 \to 108R_1 - 9R_3$$

$$\sim \begin{pmatrix} 10692 & 0 & 0 & 10692 \\ 0 & -10692 & 0 & -10692 \\ 0 & 0 & 108 & 108 \end{pmatrix} \quad R_2 \to 108R_2 + 9R_3$$

$\therefore 10692x = 10692$

$x = 1$

$\therefore -10692y = -10692$

$y = 1$

$\therefore 108z = 108$

$z = 1$

$\therefore x = y = z = 1$

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3. Solve the following system by Gauss - Jordan method

$x_1 + x_2 + x_3 + 4x_4 = -6$, $x_1 + 7x_2 + x_3 + x_4 = 12$,

$x_1 + x_2 + 6x_3 + x_4 = -5$, $5x_1 + x_2 + x_3 + x_4 = 4$.

Solution:

The given system of equation can be written in matrix form $AX = B$.

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 \\ 1 & 1 & 6 & 1 \\ 5 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -6 \\ 12 \\ -5 \\ 4 \end{pmatrix}$$

$\therefore$ The augmented matrix is

$$(A/B) = \begin{pmatrix} 1 & 1 & 1 & 4 & -6 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 5 & 1 & 1 & 1 & 4 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & -18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & -4 & -4 & -19 & 34 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - 5R_1 \end{matrix}$$

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$$\sim \begin{pmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & -24 & -126 & 276 \end{pmatrix} \quad R_4 \to R_4 + 4R_1$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & -702 & 1404 \end{pmatrix} \quad R_4 \to 5R_4 + 24R_3$$

$$\sim \begin{pmatrix} 6 & 0 & 6 & 27 & -54 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & -702 & 1404 \end{pmatrix} \quad R_1 \to 6R_1 - R_2$$

$$\sim \begin{pmatrix} 30 & 0 & 0 & 153 & -276 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & -702 & 1404 \end{pmatrix} \quad R_1 \to 5R_1 - 6R_3$$

$$\sim \begin{pmatrix} 30 & 0 & 0 & 153 & -276 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 3510 & 0 & -3510 \\ 0 & 0 & 0 & -702 & 1404 \end{pmatrix} \quad R_3 \to 702R_3 - 3R_4$$

$$\sim \begin{pmatrix} 21060 & 0 & 0 & 0 & 21060 \\ 0 & -6 & 0 & -3 & 18 \\ 0 & 0 & 3510 & 0 & -3510 \\ 0 & 0 & 0 & -702 & 1404 \end{pmatrix} \quad R_1 \to 702R_1 + 153R_4$$

$$\sim \begin{pmatrix} 21060 & 0 & 0 & 0 & 21060 \\ 0 & 4212 & 0 & 0 & 8424 \\ 0 & 0 & 3510 & 0 & -3510 \\ 0 & 0 & 0 & -702 & 1404 \end{pmatrix} \quad R_2 \to 702R_2 - 3R_4$$

$\therefore 21060\, x_1 = 21060$

$x_1 = 1$

$4212\, x_2 = 8424$

$x_2 = 2$

$3510\, x_3 = -3510$

$x_3 = -1$

$-702\, x_4 = 1404$

$x_4 = -2$