TOPIC : Gauss Elimination method

Solution of linear System

Gaussian Elimination method (or) Gauss-Elimination method:

Problems:

1 Apply the Gauss Elimination method to find the Solution of the following System.

$$2x + 3y - z = 5, \quad 4x + 4y - 3z = 3, \quad 2x - 3y + 2z = 2$$

Solution:

The given system of equations can be written in matrix form $AX = B$

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 2 & -3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$$

$\therefore$ The augmented matrix is

$$(A/B) \sim \begin{pmatrix} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 2 & 2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3/2 & -1/2 & 5/2 \\ 4 & 4 & -3 & 4 \\ 2 & -3 & 2 & 2 \end{pmatrix} \quad R_1 \longrightarrow R_1/2$$

$$\sim \begin{pmatrix} 1 & 3/2 & -1/2 & 5/2 \\ 0 & 10 & -7 & -1 \\ 0 & -6 & 3 & -3 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_3 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \begin{pmatrix} 1 & 3/2 & -1/2 & 5/2 \\ 0 & 10 & -7 & -1 \\ 0 & 0 & -12 & -36 \end{pmatrix} \quad R_3 \to 10R_3 + 6R_2$$

Use back substitution to find the solution to the system.

$$-12z = -36$$
$$z = \frac{-36}{-12}$$
$$z = 3$$

$$10y - 7z = -1$$
$$10y - 7(3) = -1$$
$$10y = -1 + 21$$
$$y = 2$$

$$x + 3/2\, y - \tfrac{1}{2} z = 5/2$$
$$x + 3/2(2) - \tfrac{1}{2}(3) = 5/2$$
$$x + 3 - 3/2 = 5/2$$
$$x = 5/2 + 3/2 - 3$$
$$\therefore x = 1$$

$\therefore x = 1,\ y = 2,\ z = 3$

2. Solve the system of equations $x_1 - x_2 + x_3 = 1$, $-3x_1 + 2x_2 - 3x_3 = -6$, $2x_1 - 5x_2 + 4x_3 = 5$. by using Gauss Elimination method

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Solution:

The given system of equations can be written in matrix form $AX = B$

$$\begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -3 \\ 2 & -5 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -6 \\ 5 \end{pmatrix}$$

∴ The augmented matrix is

$$(A/B) \sim \begin{pmatrix} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & -3 & 2 & 3 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 12 \end{pmatrix} \quad R_3 \to R_3 - 3R_2$$

Use back substitution to find the solution to the system

$$\therefore 2x_3 = 12$$
$$x_3 = 6$$

$$-x_2 + 0z = -3$$
$$-x_2 = -3$$
$$x_2 = 3$$
$$x_1 - x_2 + x_3 = 1$$
$$6 - 3 + x_3 = 1 \qquad \therefore x_3 = -2$$

3. Solve the system of equations by Gauss-Elimination method $x_1+x_2+x_3+x_4=2$, $x_1+x_2+3x_3-2x_4=-6$, $2x_1+3x_2-x_3+2x_4=7$, $x_1+2x_2+x_3-x_4=-2$

Solution:

The given system of equation can be written in matrix form $AX=B$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & -2 \\ 2 & 3 & -1 & 2 \\ 1 & 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ 7 \\ -2 \end{pmatrix}$$

$\therefore$ The augmented matrix is

$$(A/B) \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 3 & -2 & -6 \\ 2 & 3 & -1 & 2 & 7 \\ 1 & 2 & 1 & -1 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 1 & 0 & -2 & -4 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 3 & -2 & -7 \end{pmatrix} \quad R_4 \to R_4 - R_3$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 0 & 0 & 5 & 10 \end{pmatrix} \quad R_2 \leftrightarrow R_3$$

Use back substitution to find the solution to the system

$\therefore 5x_4 = 10$

$x_4 = 2$

(10)

$2x_3 - 3x_4 = -8$

$2(x_3) - 3(2) = -8$

$2x_3 = -8+6$

$x_3 = -1$

$\therefore x_2 - 3x_3 + 0x_4 = 3$

$x_2 - 3(-1) + 0 = 3$

$x_2 = 0$

$\therefore x_1 + x_2 + x_3 + x_4 = 2$

$x_1 + 0 - 1 + 2 = 2$

$x_1 + 1 = 2 - 2 + 1$

$x_1 = 1$

$\therefore x_1 = 0, \quad x_2 = 0, \quad x_3 = -1, \quad x_4 = 2$

$$\Rightarrow \begin{vmatrix} 2zp-y & \frac{z-xp}{z^2} \\ 2zq-x & -\frac{xq}{z^2} \end{vmatrix} = 0$$

$$(2zp-y)\left(-\frac{xq}{z^2}\right) - (2zq-x)\left(\frac{z-xp}{z^2}\right) = 0$$

$$\frac{1}{z^2}\left[-\cancel{2zp}xq + yxq - 2zqz + \cancel{2zq}xp + xz - x^2p\right] = 0$$

$$xyq + xz - 2z^2q - x^2p = 0$$

$$xz = 2z^2q + x^2p - xyq$$

$$xz = x^2p - q(xy - 2z^2)$$

4. $z = f(x^2+y^2)$.

Sol:

$$z = f(x^2+y^2)$$

Diff p.w.r.to x, we get

$$\frac{\partial z}{\partial x} = f'(x^2+y^2)\cdot 2x$$

$$p = 2x\, f'(x^2+y^2)$$

Diff p.w.r.to y, we get

$$\frac{\partial z}{\partial y} = f'(x^2+y^2)\cdot 2y$$

$$q = 2y\, f'(x^2+y^2)$$

$$\frac{p}{q} = \frac{\cancel{2}x\, \cancel{f'(x^2+y^2)}}{\cancel{2}y\,\cancel{f'(x^2+y^2)}}$$

$$\frac{p}{q} = \frac{x}{y}$$

$$py - qx = 0.$$