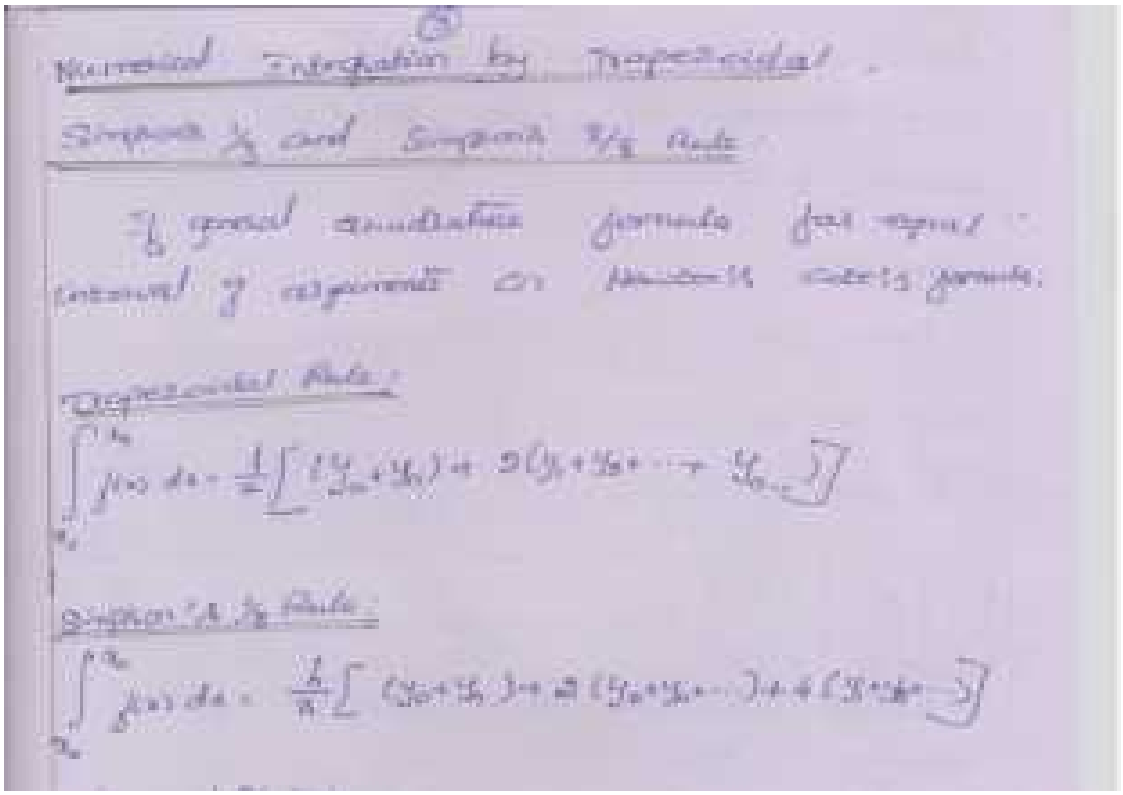
TOPIC : 3.5 Numerical single integration by Trapezoidal rule



Numerical Integration by Trapezoidal

Simpson's 1/3 and Simpson's 3/8 Rule

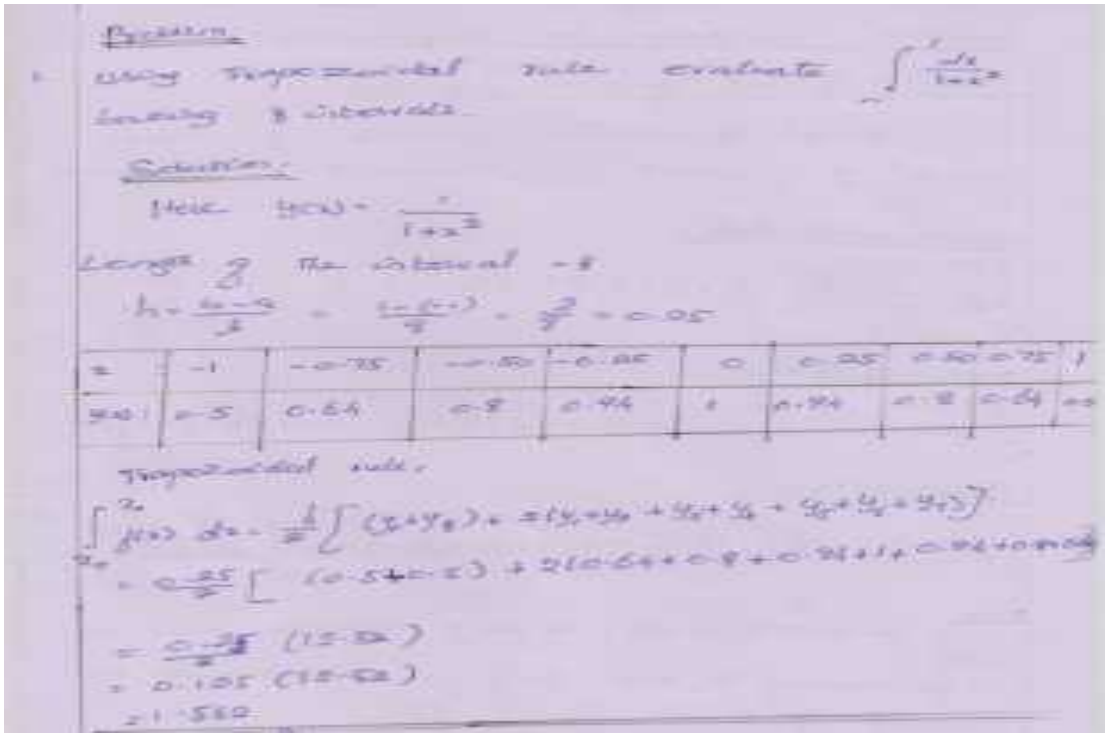
To find quadrature formula for equal interval of argument or Newton-Cotes formula.

Trapezoidal Rule:

$$\int_{x_0}^{x_n} y(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

Simpson's 1/3 Rule:

$$\int_{x_0}^{x_n} y(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots)+2(y_2+y_4+\cdots)\right]$$



Problem

1. Using Trapezoidal rule evaluate $\int_0^1 \frac{dx}{1+x^2}$ taking 8 intervals.

Solution:

Here $y(x) = \frac{1}{1+x^2}$

Length of the interval $= 1$

$h = \frac{b-a}{n} = \frac{1-0}{8} = \frac{1}{8} = 0.125$

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
y(x)	1	0.9846	0.9412	0.8767	0.8	0.7191	0.64	0.5663	0.5

Trapezoidal rule:

$$\int_{x_0}^{x_n} y(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$= \frac{0.125}{2}\left[(1+0.5)+2(0.9846+0.9412+0.8767+0.8+0.7191+0.64+0.5663)\right]$$

$$= \frac{0.125}{2}(13.94)$$

$$= 0.125\,(6.97)$$

$$= 0.8712$$

5. Problem: $\int_{-3}^{3} x^4 dx$ by using i) Trapezoidal Rule, ii) Simpson's $\frac{1}{3}$ rule and iii) Simpson $\frac{3}{8}$ rule (iv) Actual Integration

Solution:

$$\int_{-3}^{3} x^4 dx$$

Here $f(x) = x^4$

$$h = \frac{b-a}{n} = \frac{3-(-3)}{6} = 1$$

The table is:

x	-3	-2	-1	0	1	2	3
$y=f(x)$	81	16	1	0	1	16	81

Trapezoidal Rule:

$$\int_{-3}^{3} x^4 dx = \frac{1}{2}[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)]$$

$$\int_{-3}^{3} x^4 dx = (6-0)\left[\frac{196}{2}\right] = 588$$

Simpson's $\frac{1}{3}$ rule:

$$\int_{-3}^{3} x^4 dx = \frac{1}{6}[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)]$$

$$= 0.288(294)$$

$$\int_{-3}^{3} x^4 dx = 97.920$$