



TOPIC : 3.6 Numerical single integration by Simpson's 1/3 rule

Numerical Integration by Trapezoidal
Simpson's 1/3 and Simpson's 3/8 Rule

If given continuous formula for equal interval of segments or Newton's table's formula:

Trapezoidal Rule:

$$\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

Simpson's 1/3 Rule:

$$\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots)]$$

Problem

Using Trapezoidal rule evaluate $\int_{-1}^1 \frac{dx}{1+x^2}$
using 8 intervals.

Solution:

Here $f(x) = \frac{1}{1+x^2}$
Length of the interval = 2
 $h = \frac{b-a}{n} = \frac{1-(-1)}{8} = \frac{2}{8} = 0.25$

x	-1	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1
y=f(x)	0.5	0.64	0.8	0.94	1	1.04	1.2	1.44	2

Trapezoidal rule:

$$\int_{-1}^1 f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)]$$

$$= \frac{0.25}{2} [(0.5 + 2) + 2(0.64 + 0.8 + 0.94 + 1 + 1.04 + 1.2 + 1.44)]$$

$$= \frac{0.25}{2} (15.32)$$

$$= 0.125 (15.32)$$

$$= 1.915$$



8. Evaluate $\int_{-3}^3 x^4 dx$ by using (i) Trapezoidal rule, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule (ii) Actual integration

Solution:

$$\int_{-3}^3 x^4 dx$$

Here, $f(x) = x^4$

$$h = \frac{b-a}{n} = \frac{3-(-3)}{6} = 1$$

The table is,

x	-3	-2	-1	0	1	2	3
y(x)	81	16	1	0	1	16	81

Trapezoidal rule:

$$\int_{-3}^3 x^4 dx = \frac{1}{2} \left[(81+81) + 2(16+1+0+1+16) \right]$$

$$\int_{-3}^3 x^4 dx = (0.5) \{ 285 \} = 142.5$$

Simpson's $\frac{1}{3}$ rule:

$$\int_{-3}^3 x^4 dx = \frac{1}{6} \left[(81+81) + 4(16+0+16) + 2(1+1) \right]$$

$$= 0.166(294)$$

$$\int_{-3}^3 x^4 dx = 48.966$$