TOPIC : 3 - Taylor's series Method for solving simultaneous first order differential equations

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Taylor Series Method for Simultaneous first order Differential Equations

Example: Solve the system of equations

$\frac{dy}{dx} = z - x^2, \quad \frac{dz}{dx} = y + x$ with $y(0) = 1$, $z(0) = 1$

by taking $h = 0.1$ to get $y(0.1)$ and $z(0.1)$. Here y and z are dependent variables.

Solution: $x_0 = 0, \; y_0 = 1, \; z_0 = 1$

$y' = z - x^2$	$y_0' = z_0 - x_0^2 = 1 - 0 = 1$	$z' = x + y$	$z_0' = x_0 + y_0 = 0 + 1 = 1$
$y'' = z' - 2x$	$y_0'' = z_0' - 2x_0 = 1 - 2(0) = 1$	$z'' = 1 + y'$	$z_0'' = 1 + y_0' = 1 + 1 = 2$
$y''' = z'' - 2$	$y_0''' = z_0'' - 2 = 2 - 2 = 0$	$z''' = y''$	$z_0''' = y_0'' = 1$
$y^{iv} = z'''$	$y_0^{iv} = z_0''' = 1$	$z^{iv} = y'''$	$z_0^{iv} = y_0''' = 0$

By Taylor Series for y_1 and z_1 we have.

$$y_1 = y(0.1) = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots$$

$$= 1 + \frac{(0.1)}{1!}(1) + \frac{(0.1)^2}{2!}(1) + \frac{(0.1)^3}{3!}(0) + \frac{(0.1)^4}{4!}(1) + \cdots$$

$$= 1 + 0.1 + 0.005 + 0 + 0.0000042$$

$$= 1.1050$$

$$z_1 = z(0.1) = z_0 + \frac{h}{\lfloor 1}z_0' + \frac{h^2}{\lfloor 2}z_0'' + \frac{h^3}{\lfloor 3}z_0''' + \frac{h^4}{\lfloor 4}z_0^{iv} + \cdots$$

$$= 1 + (0.1)(1) + \frac{(0.1)^2}{\lfloor 2}(2) + \frac{(0.1)^3}{\lfloor 3}(1) + \frac{(0.1)^4}{\lfloor 4}(0) + \cdots$$

$$= 1 + 0.1 + 0.01 + 0.000167$$

$$= 1.110167$$

<u>Solving</u> <u>Higher</u> <u>order</u> <u>Linear Differential</u> <u>Equations</u>.

<u>Example</u>: By Taylor's series Method find $y(0.1)$ given that $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$.

<u>Solution</u>: Here $x_0 = 0$, $y_0 = 1$, $y_0' = 0$.

Given $y'' = y + xy'$

$y''' = y' + xy'' + y'$

$= 2y' + xy''$

$y^{iv} = 2y'' + xy''' + y''$

$= 3y'' + xy'''$

$y_0'' = y_0 + x_0 y_0'$

$= 1 + (0)(0) = 1$

$y_0''' = 2y_0' + x_0 y_0''$

$= 2(0) + 0(1) = 0$

$y_0^{iv} = 3y_0'' + x_0 y_0'''$

$= 3(1) + (0)(0) = 3$.

$$\therefore\ y(x) = y_0 + x y_0' + \frac{x^2}{\lfloor 2} y_0'' + \frac{x^3}{\lfloor 3} y_0''' + \cdots$$

$$= 1 + 0 + \frac{x^2}{\lfloor 2}(1) + 0 + \frac{x^4}{\lfloor 4}(3) + \cdots$$

$$= 1 + \frac{x^2}{2} + \frac{x^4}{8} + \cdots$$

$$y(0.1) = 1 + \frac{(0.1)^2}{2} + \frac{(0.1)^4}{8}$$

$$= 1 + 0.005 + 0.0000125 = 1.0050.$$