Topic : Euler's & Modified Euler's Method

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Euler and Modified Euler Method:

Example: Using Eulers Method find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x+y$, $y(0)=1$ with $h=0.2$.

Solution: $f(x,y) = x+y$, $x_0=0$, $y_0=1$, $x_1=0.2$, $x_2=0.4$.

By Euler Algorithm.

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 1 + (0.2)\,[x_0+y_0]$$
$$y(0.2) = 1+(0.2)\,[0+1] = 1.2.$$

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 1.2 + (0.2)[x_1+y_1]$$
$$= 1.2 + 0.2\,[0.2+1.2]$$
$$y(0.4) = 1.2 + 0.28 = 1.48$$

$$y_3 = y(0.6) = y_2 + h f(x_2, y_2)$$
$$= 1.48 + 0.2\,[x_2+y_2]$$
$$= 1.48 + 0.2\,(0.4+1.48)$$
$$= 1.48 + 0.376$$
$$y(0.6) = 1.856$$

Example: Using Modified Eulers Method compute $y(0.1)$ with $h=0.1$ from $y' = y - \frac{2x}{y}$, $y(0)=1$

Solution: given $f(x,y) = y - \frac{2x}{y}$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$

$h = 0.1$

By Modified Euler Method.

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$= 1 + (0.1)\left[f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\left(y_0 - \frac{2x_0}{y_0}\right)\right)\right]$$

$$= 1 + (0.1)\, f\left[0.05, 1 + (0.05)\,[1 - 0]\right]$$

$$= 1 + (0.1)\, f(0.05, 1.05)$$

$$= 1 + (0.1)\,[1.05 - 0.0952]$$

$$= 1 + (0.09548)$$

$$y_1 = 1.09548.$$

Example: Using Modified Euler Method find $y(0.1)$ if $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$

Solution: given $f(x,y) = x^2 + y^2$, $x_0 = 0$, $y_0 = 1$

$h = 0.1$, $x_1 = 0.1$

By Modified Euler Method.

$$y_{n+1} = y_n + h\, f\left[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right]$$

$$y_1 = y_0 + h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)$$

$$f(x_0, y_0) = x_0^2 + y_0^2 = 0 + 1 = 1$$

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$$y_1 = 1+(0.1)\,f\left[0+\tfrac{0.1}{2},\ 1+\tfrac{0.1}{2}(1)\right]$$
$$= 1+(0.1)\,f(0.05, 1.05)$$
$$= 1+(0.1)\,(1.105)$$
$$= 1.1105$$
$$y(0.1) = 1.1105.$$

Fourth order Runge Kutta Method for Solving first and second order Equations

Second order RK method.

If the initial values of (x,y) for the differential equation $\frac{dy}{dx} = f(x,y)$ then the first increment in y namely Δy is calculated from the formula.

$$K_1 = h\,f(x,y)$$
$$K_2 = h\,f\left(x+\frac{h}{2},\ y+\frac{K_1}{2}\right)$$
$$\Delta y = K_2 \quad \text{where } h = \Delta x.$$

Third order RK method

$$K_1 = h\,f(x,y)$$
$$K_2 = h\,f\left(x+\frac{h}{2},\ y+\frac{K_1}{2}\right)$$
$$K_3 = h\,f(x+h,\ y+2K_2-K_1)$$

and $\Delta y = \frac{1}{6}(K_1+4K_2+K_3)$