Topic : Milne's Predictor & Corrector Method

$y_1 = y_0 + \Delta y$

$= 0 + 0.34482$

$\boxed{y(0.3) = 0.34482}$

$z_1 = z_0 + \Delta z$

$z(0.3) = 1 - 0.01011$

$\boxed{z(0.3) = 0.98989}$

Milne's Predictor and Corrector Methods

Example: given $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$. The values of $y(0.2) = 2.073$, $y(0.4) = 2.452$, and $y(0.6) = 3.023$ are got by RK method of fourth order. Find $y(0.8)$ by Milne's predictor and corrector method taking $h = 0.2$.

Solution. Here

$x_0 = 0$, $y_0 = 2$

$x_1 = 0.2$, $y_1 = 2.073$

$x_2 = 0.4$, $y_2 = 2.452$

$x_3 = 0.6$, $y_3 = 3.023$.

$y' = f(x, y) = x^3 + y$.

By Milne's predictor formula.

$$y_{n+1,P} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$y_{4,P} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

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Given $y' = x^3 + y$

$$y_1' = x_1^3 + y_1 = (0.2)^3 + 2.073 = 2.081$$

$$y_2' = x_2^3 + y_2 = (0.4)^3 + 2.452 = 2.516$$

$$y_3' = x_3^3 + y_3 = (0.6)^3 + 3.023 = 3.239$$

$$y_{4,p} = 2 + \frac{4(0.2)}{3}\left[2(2.081) - 2.516 + 2(3.239)\right]$$

$$= 2 + \frac{0.8}{3}(8.124)$$

$$= 4.1664.$$

Using Milne's corrector Method.

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1}'\right]$$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + y_3' + y_4'\right]$$

$$y_1' = x_1^3 + y_1 = (0.2)^3 + 2.073 = 2.081$$

$$y_2' = 2.516$$

$$y_3' = 3.239$$

$$y_4' = x_4^3 + y_4 = (0.8)^3 + 4.1664 = 4.6784$$

$$y_{4,c} = 2.452 + \frac{0.2}{3}\left[2.516 + 4(3.239) + 4.6784\right]$$

$$= 2.452 + \frac{0.2}{3}(20.1504)$$

$$= 3.79536.$$

$$\boxed{y(0.8) = 3.79536}$$

Example: Determine the value of $y(0.4)$ using Milne's Method given $y' = xy + y^2$, $y(0) = 1$, use Taylor's series to get the values of $y(0.1)$, $y(0.2)$ and $y(0.3)$.

Solution: Here $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$, $x_2 = 0.2$,
$x_3 = 0.3$, $x_4 = 0.4$.

$$y' = xy + y^2$$
$$y'' = xy' + y + 2yy'$$
$$y''' = xy'' + y' + y' + 2yy'' + 2(y')^2$$
$$= xy'' + 2y' + 2yy'' + 2(y')^2$$

$$y_0' = x_0 y_0 + y_0^2 = 1$$
$$y_0'' = x_0 y_0' + y_0 + 2y_0 y_0' = 3$$
$$y_0''' = x_0 y_0'' + 2y_0' + 2y_0 y_0'' + 2(y_0')^2$$
$$= 10$$

$$y_1 = y_0 + h y_0' + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + \cdots$$
$$= 1 + (0.1)(1) + \frac{0.01}{2}(3) + \frac{0.001}{6}(10) + \cdots$$
$$= 1 + 0.1 + 0.015 + 0.001666.$$

$y(0.1) = 1.1167$

$$y_1' = x_1 y_1 + y_1^2 = (0.1)(1.1167) + (1.1167)^2 = 1.3587$$
$$y_1'' = x_1 y_1' + y_1 + 2y_1 y_1'$$
$$= (0.1)(1.3587) + 1.1167 + 2(1.1167)(1.3587)$$
$$= 4.3871.$$

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$y_1''' = x_1 y_1'' + 2y_1' + 2y_1 y_1'' + 2(y_1')^2$

$= 16.4131$

$\therefore \; y_2 = y_1 + \frac{h}{\lfloor 1} y_1' + \frac{h^2}{\lfloor 2} y_1'' + \frac{h^3}{\lfloor 3} y_1''' + \cdots$

$= 1.1167 + \frac{0.1}{\lfloor 1}(1.3587) + \frac{0.01}{\lfloor 2}(4.2871) + \frac{0.001}{6}(16.4131)$

$y(0.2) = 1.2767$

$y_2' = x_2 y_2 + y_2^2 = (0.2)(1.2767) + (1.2767)^2 = 1.8853$

$y_2'' = x_2 y_2' + y_2 + 2y_2 y_2'$

$= (0.2)(1.8853) + 1.2767 + 2(1.2767)(1.8853)$

$= 6.4677.$

$y_2''' = x_2 y_2'' + 2y_2'' + 2[y_2 y_2'' + (y_2')^2]$

$= 28.6875$

$y_3 = y_2 + \frac{h}{\lfloor 1} y_2' + \frac{h^2}{\lfloor 2} y_2'' + \frac{h^3}{\lfloor 3} y_2''' + \cdots$

$= 1.2767 + (0.1)(1.8853) + \frac{0.01}{2}(6.4677) + \frac{0.001}{6}(28.6875)$

$y(0.3) = 1.5023.$

By milnes predictor method.

$y_{4,P} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3']$

$y_{4,P} = 1 + \frac{4(0.1)}{3}[2(1.3587) - 1.8853 + 2(2.7676)]$

$= 1.83297$

Now $y'_{4,P} = x_4 y_4 + y_4^2$

$= (0.4)(1.83297) + (1.83297)^2$

$= 4.09296$

Using Milne's Corrector Method.

$y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y'_{4,P}\right]$

$= 1.2767 + \frac{0.1}{3}\left[1.8853 + 4(2.7076) + 4.09296\right]$

$= 1.83698$

$y(0.4) = 1.83698$.

HW Example: Using Runge Kutta method Calculate $y(0.1)$, $y(0.2)$ and $y(0.3)$ given that $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 1$, $y(0) = 0$ Taking these values as starting values find $y(0.4)$ by Milne's method.

Adam's Predictor and Corrector Method

Example: Given $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$ evaluate $y(1.4)$ by Adam's Bashforth method.

Solution $x_0 = 1$, $x_1 = 1.1$, $x_2 = 1.2$, $x_3 = 1.3$, $x_4 = 1.4$

$y_0 = 1$, $y_1 = 1.233$, $y_2 = 1.548$, $y_3 = 1.979$, $y_4 = ?$