



TOPIC: 7 GAUSS –SEIDEL METHOD

ITERATIVE METHOD

GAUSS - SEIDEL METHOD

Formula:

$$x = \frac{1}{a_{11}} [C_1 - a_{12}y - a_{13}z]$$

$$y = \frac{1}{a_{22}} [C_2 - a_{21}x - a_{23}z]$$

$$z = \frac{1}{a_{33}} [C_3 - a_{31}x - a_{32}y]$$

Problem:

Solve the following equation by Gauss Seidel method
 $4x + 2y + z = 14$, $x + 5y - z = 10$, $x + y + 3z = 20$

Solution:

First check the given Simultaneous equation is diagonally dominant

$$|4| > |2| + |1|$$

$$|5| > |1| + |1|$$

$$|8| > |1| + |1|$$

∴ The given system of equation satisfy the diagonally dominant

Gauss - Seidel Method formula,

$$x = \frac{1}{a_{11}} [C_1 - a_{12}y - a_{13}z]$$

$$y = \frac{1}{a_{22}} [C_2 - a_{21}x - a_{23}z]$$

$$z = \frac{1}{a_{33}} [C_3 - a_{31}x - a_{32}y]$$

First iteration:

putting $y=0$ and $z=0$

$$x = \frac{1}{4} [14 - 2(0) - 1(0)] = 3.5$$

putting $x=3.5$ and $z=0$

$$y = \frac{1}{5} [10 - 1(3.5) - 1(0)] = 1.3$$

putting $x=3.5$ and $y=1.3$

$$z = \frac{1}{8} [20 - 1(3.5) - 1(1.3)] = 1.9$$

Second iteration:

putting $y=1.3$ and $z=1.9$

$$x = \frac{1}{4} [14 - 2(1.3) - 1(1.9)] = 2.316$$

putting $x=2.316$ and $z=1.9$

$$y = \frac{1}{5} [10 - 1(2.316) - 1(1.9)] = 1.905$$

putting $x=2.316$ and $y=1.905$

$$z = \frac{1}{8} [20 - 1(2.316) - 1(1.905)] = 1.905$$



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Third iteration:

put $y = 1.905$ and $z = 1.965$

$$x = \frac{1}{4} [14 - 2(1.905) - 1(1.965)] = 2.056$$

put $x = 2.056$ and $z = 1.965$

$$y = \frac{1}{5} [10 - 2.056 - (-1)(1.965)] = 1.982$$

put $x = 2.056$ and $y = 1.982$

$$z = \frac{1}{8} [20 - 2.056 - 1.982] = 1.995$$

Fourth iteration:

put $y = 1.982$ and $z = 1.995$

$$x = \frac{1}{4} [14 - 2(1.982) - 1(1.995)] = 2.010$$

put $x = 2.010$ and $z = 1.995$

$$y = \frac{1}{5} [10 - 2.010 - (-1)(1.995)] = 1.997$$

put $y = 1.997$ and $x = 2.010$

$$z = \frac{1}{8} [20 - 2.010 - 1.997] = 1.999$$

Fifth iteration:

put $y = 1.997$ and $z = 1.999$

$$x = \frac{1}{4} [14 - 2(1.997) - 1(1.999)] = 2.002$$

put $x = 2.002$ and $z = 1.999$

$$y = \frac{1}{5} [10 - 2.002 + 1.999] = 1.999$$

put $x = 2.002$ and $z = 1.999$

$$z = \frac{1}{8} [20 - 2.002 - 1.999] = 2$$

Sixth iteration:

put $y = 1.999$ and $z = 2$

$$x = \frac{1}{4} [14 - 2(1.999) - 1(2)] = 2.001$$

put $x = 2.001$ and $z = 2$



$$y = \frac{1}{5} [10 - 2.001 - (-1)(2)] = 1.9998$$

put $x = 2.001$ and $y = 1.9998$

$$z = \frac{1}{8} [20 - 2.001 - 1.9998] = 2$$

After sixth iteration, we get

$$x = 2.001, y = 1.9998, z = 1.9999 \text{ (i) } 2$$

2. Solve the following system of equation using Gauss-Seidel iteration method. $28x + 4y - z = 32$,
 $2x + 3y + 10z = 24$, $2x + 17y + 4z = 35$.

Solution:

First check the given simultaneous equations is diagonally dominant.

$$|28| > |4| + |1|$$

$$|8| > |1| + |10|$$

$$|4| < |2| + |17|$$

$\therefore$ The above equations are not satisfy diagonally dominant.

$\therefore$ We interchange second and third equations

$$28x + 4y - z = 32$$

$$2x + 17y + 4z = 35$$

$$2x + 3y + 10z = 24$$

$$|28| > |4| + |1|$$

$$|17| > |2| + |4|$$

$$|10| > |1| + |3|$$

$\therefore$ The above equations are diagonally dominant.



Gauss - Seidel formula is,

$$x = \frac{1}{a_{11}} [C_1 - a_{12}y - a_{13}z]$$

$$y = \frac{1}{a_{22}} [C_2 - a_{21}x - a_{23}z]$$

$$z = \frac{1}{a_{33}} [C_3 - a_{31}x - a_{32}y]$$

First iteration:

put $y=0$ and $z=0$

$$x = \frac{1}{28} [32 - 4y + (1)z] = \frac{1}{28} [32 - 4(0) + 0] = 1.143$$

put $x=1.143$ and $z=0$

$$y = \frac{1}{17} [35 - 2x - 4z] = \frac{1}{17} [35 - 2(1.143) - 4(0)] = 1.924$$

put $x=1.143$ and $y=1.924$

$$z = \frac{1}{10} [24 - x - 3y] = \frac{1}{10} [24 - (1.143) - 3(1.924)] = 1.709$$

Second iteration:

put $y=1.924$, $z=1.709$

$$x = \frac{1}{28} [32 - 4y + (1)z] = \frac{1}{28} [32 - 4(1.924) + 1.709] = 0.929$$

put $x=0.929$, $z=1.709$

$$y = \frac{1}{17} [35 - 2(0.929) - 4(1.709)] = 1.547$$

put $x=0.929$, $y=1.547$.

$$z = \frac{1}{10} [24 - 0.929 - 3(1.547)] = 1.843$$

Third iteration:

put $y=1.547$ and $z=1.843$

$$x = \frac{1}{28} [32 - 4(1.547) + 1.843] = 0.988$$

put $x=0.988$ and $z=1.843$

$$y = \frac{1}{17} [35 - 2(0.988) - 4(1.843)] = 1.509$$

put $x=0.988$ and $y=1.509$



$$Z = \frac{1}{10} [24 - 0.988 - 3(1.509)] = 1.849.$$

Fourth iteration:

$$\text{Put } y = 1.509 \text{ and } Z = 1.849$$

$$x = \frac{1}{28} [32 - 4(1.509) + 1.849] = 0.993$$

$$\text{Put } x = 0.993 \text{ and } Z = 1.849$$

$$y = \frac{1}{17} [35 - 2(0.993) - 4(1.849)] = 1.506.$$

$$\text{put } x = 0.993 \text{ and } y = 1.506.$$

$$Z = \frac{1}{10} [24 - x - 3y] = \frac{1}{10} [24 - 0.993 - 3(1.506)]$$

Fifth iteration:

$$\text{put } y = 1.506 \text{ and } Z = 1.849$$

$$x = \frac{1}{28} [32 - 4(1.506) + 1.849] = 0.994$$

$$\text{put } x = 0.994 \text{ and } Z = 1.849$$

$$y = \frac{1}{17} [35 - 2(0.994) - 4(1.849)] = 1.507$$

$$\text{put } x = 0.994 \text{ and } y = 1.507$$

$$Z = \frac{1}{10} [24 - 0.994 - 3(1.507)] = 1.849$$

Sixth iteration:

$$\text{put } y = 1.507 \text{ and } Z = 1.849$$

$$x = \frac{1}{28} [32 - 4(1.507) + 1.849] = 0.994$$

$$\text{put } x = 0.994 \text{ and } Z = 1.849$$

$$y = \frac{1}{17} [35 - 2(0.994) - 4(1.849)] = 1.507$$

$$\text{put } x = 0.994 \text{ and } y = 1.507$$

$$Z = \frac{1}{10} [24 - 0.994 - 3(1.507)] = 1.849.$$

$$\therefore x = 0.994, \quad y = 1.507 \text{ and } Z = 1.849$$