TOPIC : 10 – Matrix inversion by gauss jordan method

INVERSE OF A MATRIX BY GAUSS JORDAN METHOD:

Problems:

1. Find the inverse of the matrix $\begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ by Gauss-Jordan method.

Solution:

Let $AX = I$

where $A = \begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$

∴ The augmented matrix is

$$(A/I) \sim \begin{pmatrix} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 5 & -2 & 1 & 0 \\ 0 & 26 & -3 & 5 \end{pmatrix} \quad R_2 \rightarrow 5R_2 - 3R_1$$

$$\sim \begin{pmatrix} 130 & 0 & 20 & 10 \\ 0 & 26 & -3 & 5 \end{pmatrix} \quad R_1 \rightarrow 26R_1 + 2R_2$$

$$\sim \begin{pmatrix} 1 & 0 & 20/130 & 10/130 \\ 0 & 1 & -3/26 & 5/26 \end{pmatrix} \quad \begin{matrix} R_1 \rightarrow R_1/130 \\ R_2 \rightarrow R_2/26 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 2/13 & 1/13 \\ 0 & 1 & -3/26 & 5/26 \end{pmatrix}$$

Hence the inverse of the given matrix is

$$\begin{pmatrix} 2/13 & 1/13 \\ -3/26 & 5/26 \end{pmatrix} = 1/26 \begin{pmatrix} 4 & 2 \\ -3 & 5 \end{pmatrix}$$

2. Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{pmatrix}$ using Gauss - Jordan method.

Solution:

Let $AX = I$

where $A = \begin{pmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{pmatrix}$, $X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

∴ The augmented matrix is

$$(A/I) \sim \begin{pmatrix} 1 & 2 & 6 & 1 & 0 & 0 \\ 2 & 5 & 15 & 0 & 1 & 0 \\ 6 & 15 & 46 & 0 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 3 & 10 & -6 & 0 & 1 \end{pmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 6R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 1 \end{pmatrix} \; R_3 \to R_3 - 3R_2$$

$$\sim \begin{pmatrix} 1 & 2 & 0 & 1 & 18 & -6 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 1 \end{pmatrix} \; R_1 \to R_1 - 6R_3$$

2 Find the largest Eigenvalue and the corresponding Eigenvector $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ find also the least Eigenvalue and hence find the 3rd Eigenvalue also. (or) Using power method find all the Eigenvalues.

Solution:

Let $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Let $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an arbitary Eigenvalue.

$$AX_1 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0+0 \\ 1+0+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1X_2$$

$$AX_2 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+6+0 \\ 1+2+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = 7\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = 7X_3$$

$$AX_3 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+2.5716+0 \\ 1+0.8572+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 3.5716 \\ 1.8572 \\ 0 \end{pmatrix}$$

$$= 3.5716\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = 3.5716X_4$$

$$AX_4 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+3.12+0 \\ 1+1.04+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 4.12 \\ 2.04 \\ 0 \end{pmatrix} =$$

$$4.12\begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} = 4.12X_5$$

Problems:

1. Find the Numerically largest eigenvalue of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ also find the corresponding Eigenvector.

Solution:

$$A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

Let $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be the arbitrary Eigenvector.

$$AX_1 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 25 \\ 1 \\ 2 \end{pmatrix} = 25\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix}$$

$$AX_2 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = \begin{pmatrix} 25.2 \\ 1.12 \\ 1.68 \end{pmatrix} = 25.2\begin{pmatrix} 1 \\ 0.044 \\ 0.0667 \end{pmatrix}$$

$$AX_3 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.044 \\ 0.0667 \end{pmatrix} = \begin{pmatrix} 25.177 \\ 1.132 \\ 1.733 \end{pmatrix} = 25.177\begin{pmatrix} 1 \\ 0.045 \\ 0.0688 \end{pmatrix}$$

$$AX_4 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.045 \\ 0.0688 \end{pmatrix} = \begin{pmatrix} 25.1826 \\ 1.135 \\ 1.7248 \end{pmatrix} = 25.1826\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$$

$$AX_5 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1821 \\ 1.135 \\ 1.726 \end{pmatrix} = 25.1821\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$$

$\therefore$ The largest Eigenvalue is 25.18121

$\therefore$ The corresponding Eigenvector is $\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$

2 Find the largest Eigenvalue and the corresponding Eigenvector $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ find also the least Eigenvalue and hence find the 3rd Eigenvalue also. (or) Using power method find all the Eigenvalues.

Solution:

Let $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Let $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be an arbitary Eigenvalue.

$$AX_1 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0+0 \\ 1+0+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1X_2$$

$$AX_2 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+6+0 \\ 1+2+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = 7\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = 7X_3$$

$$AX_3 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+2.5716+0 \\ 1+0.8572+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 3.5716 \\ 1.8572 \\ 0 \end{pmatrix}$$

$$= 3.5716\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = 3.5716X_4$$

$$AX_4 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+3.12+0 \\ 1+1.04+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 4.12 \\ 2.04 \\ 0 \end{pmatrix} =$$

$$4.12\begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} = 4.12X_5$$

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$$Ax_5 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+2.9706+0 \\ 1+0.9902+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 3.9706 \\ 1.9902 \\ 0 \end{pmatrix}$$

$$= 3.9706 \begin{pmatrix} 1 \\ 0.5012 \\ 0 \end{pmatrix} = 3.9706 x_6$$

$$Ax_6 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.5012 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+3.0072+0 \\ 1+1.0024+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 4.0072 \\ 2.0024 \\ 0 \end{pmatrix}$$

$$= 4.0072 \begin{pmatrix} 1 \\ 0.4997 \\ 0 \end{pmatrix} = 4.0072 x_7$$

$$Ax_7 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.4997 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+2.9982+0 \\ 1+0.9994+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 3.9982 \\ 1.9994 \\ 0 \end{pmatrix}$$

$$= 3.9982 \begin{pmatrix} 1 \\ 0.500 \\ 0 \end{pmatrix} = 3.9982 X_8$$

$$Ax_8 = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.500 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+3+0 \\ 1+1+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$$

$$= 4 x_9$$

∴ The largest Eigen Value of $A = 4$ and their Corresponding Eigenvector is $\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$.

<u>To find the least Eigen Value of A:</u>

Let $B = A - \lambda_1 I$

$B = A - 4I$ [Since $\lambda_1 = 4$]

$$= \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} - 4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Let $Y_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be the arbitary Eigen value of B.

$$BY_1 = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3+0+0 \\ 1+0+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$$

$$= -3 \begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -3Y_2$$

$$BY_2 = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = \begin{pmatrix} -3-1.9998+0 \\ 1+0.6666+0 \\ 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} -4.9998 \\ 1.6666 \\ 0 \end{pmatrix} = -4.9998 \begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -4.9998 Y_3$$

$$BY_3 = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 1.6666 \\ 0 \end{pmatrix} = -5 \begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix}$$

∴ The largest Eigen Value of B is -5 and their Corresponding Eigenvector $\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix}$

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Smallest Eigenvalue of A = Largest Eigenvalue of A + Largest Eigenvalue of B

$= 4 + (-5)$

$= -1$