TOPIC : 3 - Taylor's series Method for solving higher order differential equations

$$z_1 = z(0.1) = z_0 + \frac{h}{|1} z_0' + \frac{h^2}{|2} z_0'' + \frac{h^3}{|3} z_0''' + \frac{h^4}{|4} z_0^{iv} + \cdots$$

$$= 1 + (0.1)(1) + \frac{(0.1)^2}{|2}(2) + \frac{(0.1)^3}{|3}(1) + \frac{(0.1)^4}{|4}(0) + \cdots$$

$$= 1 + 0.1 + 0.01 + 0.000167$$

$$= 1.110167$$

Solving Higher order Linear Differential Equations.

Example: By Taylor's series Method find $y(0.1)$ given that $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$.

Solution: Here $x_0 = 0$, $y_0 = 1$, $y_0' = 0$.

Given $y'' = y + xy'$

$$y''' = y' + xy'' + y' = 2y' + xy''$$

$$y^{iv} = 2y'' + xy''' + y'' = 3y'' + xy'''$$

$$y_0'' = y_0 + x_0 y_0' = 1 + (0)(0) = 1$$

$$y_0''' = 2y_0' + x_0 y_0'' = 2(0) + 0(1) = 0$$

$$y_0^{iv} = 3y_0'' + x_0 y_0''' = 3(1) + (0)(0) = 3$$

$$\therefore y(x) = y_0 + x y_0' + \frac{x^2}{|2} y_0'' + \frac{x^3}{|3} y_0''' + \cdots$$

$$= 1 + 0 + \frac{x^2}{|2}(1) + 0 + \frac{x^4}{|4}(3) + \cdots$$

$$= 1 + \frac{x^2}{2} + \frac{x^4}{8} + \cdots$$

$$y(0.1) = 1 + \frac{(0.1)^2}{2} + \frac{(0.1)^4}{8}$$

$$= 1 + 0.005 + 0.0000125 = 1.0050.$$