$$\Delta y = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= \frac{1}{6}\left[0.2 + 2(0.19672) + 2(1.1967) + 0.1891\right]$$

$$= 0.19598.$$

$$y(0.2) = y_1 = y_0 + \Delta y = 1 + 0.19598 = 1.19598$$

$$\boxed{y(0.2) = 1.19598}$$

RK Method for Simultaneous First order Differential Equation:

Example: Solving the system of differential equation $\frac{dy}{dx} = xz + 1$, $\frac{dz}{dx} = -xy$ for $x = 0.3$ using fourth order RK method. The initial values are $x = 0$, $y = 0$, $z = 1$.

Solution:

Given $x_0 = 0$, $y_0 = 0$, $z_0 = 1$, $h = 0.3$.

$f_1(x,y,z) = xz + 1$

$k_1 = h f(x_0, y_0, z_0)$

$= (0.3)\left[x_0 z_0 + 1\right]$

$= (0.3)\ (0 + 1)$

$= 0.3.$

$f_2(x,y,z) = -xy$

$l_1 = h f_2(x_0, y_0, z_0)$

$= (0.3)\ (-x_0 y_0)$

$= (0.3)\left(-(0)\right)$

$= 0$

(10)

$$K_2 = h f_1\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}, z_0+\frac{l_1}{2}\right)$$
$$= (0.3) f\left[0+\frac{0.3}{2}, 0+\frac{0.3}{2}, 1+\frac{0}{2}\right]$$
$$= (0.3) f_1[0.15, 0.15, 1]$$
$$= 0.345.$$

$$K_3 = h f_1\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}, z_0+\frac{l_2}{2}\right)$$
$$= (0.3) f_1\left(0.15, 0+\frac{0.345}{2}, 1+\frac{-0.007}{2}\right)$$
$$= (0.3)\left[(0.15)(0.9965)+1\right]$$
$$= 0.3448.$$

$$K_4 = h f_1(x_0+h, y_0+K_3, z_0+l_3)$$
$$= (0.3) f_1(0.03, 0.3448, 1-0.0078)$$
$$= (0.3)\left[(0.3)(0.9922)+1\right]$$
$$= 0.3893.$$

$$\Delta y = \frac{1}{6}\left[K_1+2K_2+2K_3+K_4\right]$$
$$= \frac{1}{6}[2.0689]$$
$$= 0.34482$$

$$l_2 = h f_2\left[x_0+\frac{h}{2}, y_0+\frac{K_1}{2}, z_0+\frac{l_1}{2}\right]$$
$$= (0.3) f\left[0+\frac{0.3}{2}, 0+\frac{0.3}{2}, 1+\frac{0.0}{2}\right]$$
$$= 0.3 f[0.15, 0.15, 1]$$
$$= -0.00675$$
$$= -0.007.$$

$$l_3 = h f_2\left[x_0+\frac{h}{2}, y_0+\frac{K_2}{2}, z_0+\frac{l_2}{2}\right]$$
$$= (0.3) f_2\left[0.15, 0+\frac{0.345}{2}, 1+\frac{-0.007}{2}\right]$$
$$= (0.3) f_2\left[0.15, 0.1725, 0.9965\right)$$
$$= (0.3)\left[(-0.15)(0.1725)\right]$$
$$= -0.0078.$$

$$l_4 = h f_2\left[x_0+h, y_0+K_3, z_0+l_3\right]$$
$$= (0.3) f_2(0.3, 0.3448, 0.9922]$$
$$= (0.3)\left[-(0.3)(0.3448)\right]$$
$$= -0.031032$$

$$\Delta z = \frac{1}{6}\left[l_1+2l_2+2l_3+l_4\right]$$
$$= \frac{-1}{6}[0.060632]$$
$$= -0.01011.$$