Now $y'_{4,p} = x_4 y_4 + y_4^2 =$

$= (0.4)(1.83297) + (1.83297)^2$

$= 4.09296$

Using Milne's Corrector Method.

$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y'_{4,p}]$

$= 1.2767 + \frac{0.1}{3}[1.8853 + 4(2.7076) + 4.09296]$

$= 1.83698$

$y(0.4) = 1.83698$.

H.W Example: Using Runge Kutta Method Calculate $y(0.1)$, $y(0.2)$ and $y(0.3)$ given that $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 1$, $y(0) = 0$ Taking these Values as starting values find $y(0.4)$ by Milne's Method.

Adam's Predictor and Corrector Method

Example: Given $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$ evaluate $y(1.4)$ by Adam's Bashforth Method.

Solution $x_0 = 1$, $x_1 = 1.1$, $x_2 = 1.2$, $x_3 = 1.3$, $x_4 = 1.4$

$y_0 = 1$, $y_1 = 1.233$, $y_2 = 1.548$, $y_3 = 1.979$, $y_4 = ?$

(13)

By Adam's method.

Predictor :

$$y_{n+1,P} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$$y_{4,P} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

Here $y_0' = x_0^2(1+y_0) = 1^2(1+1) = 2$

$y_1' = x_1^2(1+y_1) = (1.1)^2(1+1.233) = 2.70193$

$y_2' = x_2^2(1+y_2) = (1.2)^2(1+1.548) = 3.66912$

$y_3' = x_3^2(1+y_3) = (1.3)^2(1+1.979) = 5.0345$

$$y_{4,P} = 1.979 + \frac{0.1}{24}\left[55(5.0345) - 59(3.66912) + 37(2.70193) - 9(2)\right]$$

$$= 1.979 + \frac{0.1}{24}(142.33683)$$

$$= 2.5721$$

Corrector method :

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right]$$

$$y_{4,c} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$y_4' = x_4^2(1+y_4) = (1.4)^2[1+2.5871)$$
$$= 7.030716.$$

$$y_{4,c} = 1.979 + \frac{0.1}{24}\left[9(7.030716) + 19(5.0345) - 5(6.60912) + 2.70193\right]$$

$$= 1.979 + \frac{0.1}{24}(143.58827)$$

$$= 2.5772844$$

$$= 2.5773.$$

Example: Find $y(0.1), y(0.2), y(0.3)$ from $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$ by using R.K method and hence $y(0.4)$ using Adam's method.

Solution: $f(x,y) = xy + y^2$, $x_0 = 0, x_1 = 0.1, x_2 = 0.2$

$x_3 = 0.4$, $x_4 = 0.4$, $y_0 = 1$

$K_1 = hf(x_0, y_0) = (0.1)\, f(0.1) = (0.1)\, 1 = 0.1$

$K_2 = hf(0.05, y_0 + \frac{K_1}{2}) = (0.1)\, f(0.05, 1.05)$

$= (0.1)\left[(0.05)(1.05) + (1.05)^2\right] = 0.1155.$

$K_3 = hf\left[0.05, y_0 + \frac{K_2}{2}\right]$

$= (0.1)\, f[0.05, 1.0578)$

$= (0.1)\left[0.05(1.0578) + (1.0578)^2\right]$

$= 0.1172$

(17)

$K_4 = h\, f(x_0+h,\ y_0+K_3)$

$= (0.1)\ f(0.1,\ 1.1172)$

$= (0.1)\ [(0.1)(1.1172) + (1.1172)^2] = 0.13598$

$y(0.1) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 1.1169$

$\boxed{y(0.1) = 1.1169}$

Now. $K_1 = h\, f(x_1, y_1) = (0.1)\, f(0.1, 1.1169)$

$= 0.1359$

$K_2 = h\, f(x_1 + \frac{h}{2},\ y_1 + \frac{K_1}{2})$

$= (0.1)\, f(0.15,\ 1.1849)$

$= 0.1582$

$K_3 = h\, f(0.5,\ y_1 + \frac{K_2}{2})$

$= (0.1)\, f(0.15,\ 1.196)$

$= 0.16098$

$K_4 = h\, f(x_1+h,\ y_1 + K_3)$

$= h\, f(0.2,\ 1.2779)$

$= (0.1)\, f(0.2,\ 1.2779)$

$= 0.1889$

$y_2 = 1.1169 + \frac{1}{6}[0.1359 + 2(0.1582 + 0.16098) + 0.1889]$

$\boxed{y(0.2) = 1.2774}$

Now $K_1 = h f(x_2, y_2)$

$= (0.1) f(0.2, 1.2774) = 0.1887$

$K_2 = h f(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2})$

$= (0.1) f(0.25, 1.3178)$

$= 0.2225$

$K_3 = h f(x_2 + \frac{h}{2}, y_2 + \frac{K_2}{2})$

$= (0.1) f(0.25, 1.3887)$

$= 0.2274$

$K_4 = h f(x_3 + h, y_3 + K_3)$

$= (0.1) f(0.3, 1.5048)$

$= 0.2716$

$\Delta y = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.1887 + 2(0.2225) + 2(0.2274) + 0.2716]$

$y_3 = y_2 + \Delta y$

$= 1.5041$

Adams predictor formula:

$y_{4,P} = y_3 + \frac{h}{24}[55y_3' - 59y_2' + 37y_1' - 9y_0']$

$y_0' = x_0 y_0 + y_0^2 = 1$

(15)

$y_1' = x_1 y_1 + y_1^2 = 1.3592$

$y_2' = x_2 y_2 + y_2^2 = 1.8872$

$y_3' = x_3 y_3 + y_3^2 = 2.7135$

$\therefore y_{4,P} = 1.5041 + \frac{0.1}{24}[55(2.7135) - 59(1.8872) + 37(1.3592) - 9(1)]$

$= 1.8341$

$y'_{4,P} = x_4 y_4 + y_4^2$

$= (0.4)(1.8341) + (1.8341)^2$

$= 4.0976$

Adam's Corrector formula.

$y_{4,c} = y_3 + \frac{h}{24}[9y_4' + 19y_3' - 5y_2' + y_1']$

$= 1.5041 + \frac{0.1}{24}[9(4.0976) + 19(2.7135) - 5(1.8872) + 1.3592]$

$= 1.8389$

$\boxed{y(0.4) = 1.8389}$

HW

Example ① Using Adam's Bashforth method find $y(4.4)$ given $5xy' + y^2 = 2$, $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$.

② Find $y(0.1)$, $y(0.2)$ and $y(0.3)$ using RK method of fourth order given $\frac{dy}{dx} = \frac{1}{2}(1+x)y^2$, $y(0) = 1$. Continue your calculations to find $y(0.4)$ using Adam's Method.