16. ***Calculation of basic dimensions of the gear pair :*** Calculate all the basic dimensions of the pinion and gear using the equations listed in Table 7.1.

Note

- ✓ The above listed procedure is for the design of pinion.
- ✓ As discussed earlier, if the materials of the pinion and gear are same, then design only the pinion. If the materials of the pinion and gear are different, then design the pinion first and check for both pinion and gear.
- ✓ The induced bending stress in the gear (σ_{b2}) can be determined by using the relation

$$\sigma_{b1}\, y_{v1} = \sigma_{b2} \cdot y_{v2}$$

where σ_{b1} and σ_{b2} = Induced bending stresses of pinion and gear respectively, and

y_{v1} and y_{v2} = Form factors of pinion and gear respectively based on the virtual number of teeth.

- ✓ Since the contact area is same, the induced contact stress is same for both pinion and gear. *i.e.*, $\sigma_{c1} = \sigma_{c2}$.

③ —x—

BEVEL GEAR

Example 7.11 ***Design a cast iron bevel gear drive for a pillar drilling machine to transmit 1875 W at 800 r.p.m. to a spindle at 400 r.p.m. The gear is to work for 40 hours per week for 3 years. Pressure angle is 20°.***

Sem Model

Given Data : P = 1875 W ; N_1 = 800 r.p.m. ; N_2 = 400 r.p.m. ; $\alpha = 20°$.

% year year

$\frac{365 \times 3}{7 \text{ week}} = 156$

To find : Design a bevel gear drive.

☺ **Solution :** Since the materials of pinion and gear are same, we have to *design only the pinion.*

① ***Gear ratio :*** $i = \frac{N_1}{N_2} = \frac{800}{400} = 2$

δ (or) Delta Δ

Pitch angles : For right angle bevel gears, $\tan(\delta_2) = i = 2$ tan δ = i 8.39

or $\delta_2 = \tan^{-1}(2) = 63.43°$ $\tan^{-1} 2 = 63.43°$

and 9.39 $\delta_1 = 90° - \delta_2 = 90° - 63.43° = 26.57°$ δi = 90° − δ2

② ***Material for pinion and gear :*** Cast iron, Grade 35 heat treated.

1.40 Tensile strength stress σ_u = 350 N/mm², from Table 5.3 5.10 Page

hours weeks 3 years. 40 × 156 = 6240

156

③ ***Gear life*** in hours = (40 hrs / week) × (52 weeks / year × 3 years) = 6240 hours

Hours RPM MIN

Gear life in cycles, $N = 6240 \times 800 \times 60 = 29.952 \times 10^7$ cycles

④ ***Calculation of initial design torque $[M_t]$:***

We know that, $[M_t] = M_t \times K \times K_d$ 8.15 PSG

Bevel

$$M_t = M_t \cdot K \cdot K_d$$

$P = \frac{2\pi N_1 \cdot M_t}{60}$

where $$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 1875}{2\pi \times 800} = 22.38 \text{ N-m, and}$$

$$K \cdot K_0 = 1.3. \quad \text{... (initially assumed)}$$

$$\therefore \quad [M_t] = 22.38 \times 1.3 = 29.095 \text{ N-m}$$

5. *Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:*

Equivalent modulus 8.14 → ✓ ***To find E_{eq}:*** $E_{eq} = 1.4 \times 10^5$ N/mm² for cast iron, $\sigma_u > 280$ N/mm², from Table 5.20.

Design bending stress → ✓ ***To find $[\sigma_b]$:*** We know that the design bending stress, 8.18

$$[\sigma_b] = \frac{1.4\, K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}, \text{ for rotation in one direction} \quad 8.18$$

8.18

where 8.20 $$K_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{29.952 \times 10^7}} = 0.8852, \text{ for C.I, from Table 5.14,}$$

8.19 $K_\sigma = 1.2$, for C.I, from Table 5.15. $\Sigma\sigma$ Sigma

8.19 $n = 2$, from Table 5.17, and

8.19 $\sigma_{-1} = 0.45\, \sigma_u$.

But 140 Sigma $\sigma_u = 350$ N/mm², for C.I., from Table 5.3

$\therefore \quad \sigma_{-1} = 0.45 \times 350 = 157.5$ N/mm²

Then, $$[\sigma_b] = \frac{1.4 \times 0.8852}{2 \times 1.2} \times 157.5 = 81.33 \text{ N/mm}^2$$

✓ ***To find $[\sigma_c]$:*** We know that the design contact stress,

8.16 $$[\sigma_c] = C_B \times HB \times K_{cl}$$

where 8.16 $C_B = 2.3$, from Table 5.18,

8.16 $HB = 200$ to 260, from Table 5.18, and

8.17 $$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{29.952 \times 10^7}}$$

$$= 0.833, \text{ for C.I, from Table 5.19.}$$

$$\therefore \quad [\sigma_c] = 2.3 \times 260 \times 0.833 = 498.08 \text{ N/mm}^2$$

6. *Calculation of cone distance (R):*

Spur → Center

1st formula

PSG 8.13

We know that, $$R \geq \psi_y \sqrt{i^2 + 1}\ \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}\,[M_t]}{i}}$$

ψ Psi

Designing Bevel

25°

where 8.15 $\psi_y = R / b = 3$, initially assumed.

Eq

$$\therefore \quad R \geq 3\sqrt{2^2+1}\ \sqrt[3]{\left[\frac{0.72}{(3-0.5)\,498.08}\right]^2 \times \frac{1.4 \times 10^5 \times 17.905 \times 10^3}{2}}$$

(1) σc (1)

$$\geq 50.2$$

or $R = 51$ mm

8.22

7. Assume $z_1 = 20$; Then $z_2 = i \times z_1 = 2 \times 20 = 40$

Virtual number of teeth : 8·1 $z_{v1} = \dfrac{z_1}{\cos \delta_1} = \dfrac{20}{\cos 26.57°} \approx 23$; and

$$z_{v2} = \frac{z_2}{\cos \delta_2} = \frac{40}{\cos 63.43°} \approx 90$$

8. *Calculation of transverse module (m_t) :*

We know that, $m_t = \dfrac{R}{0.5\sqrt{z_1^2 + z_2^2}} = \dfrac{51}{0.5\sqrt{20^2+40^2}} = 2.28$ mm

From Table 5.8, the nearest higher standard transverse module is 2.5 mm

9. *Revision of cone distance (R) :*

We know that, $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 2.5\sqrt{20^2+40^2} = 55.9$ mm

10. *Calculation of b, m_{av}, d_{1av}, v and ψ_y :*

✓ Face width (b) : $b = \dfrac{R}{\psi_y} = \dfrac{55.9}{3} = 18.63$ mm

8.15 Top

✓ Average module (m_{av}) : $m_{av} = m_t - \dfrac{b \sin \delta_1}{z_1} = 2.5 - \dfrac{18.63 \times \sin 26.57°}{20}$

$$= 2.083 \text{ mm}$$

✓ Average pcd of pinion (d_{1av}) : $d_{1av} = m_{av} \times z_1 = 2.083 \times 20 = 41.66$ m

✓ Pitch line velocity (v) : $v = \dfrac{\pi \times d_{1av} \times N_1}{60} = \dfrac{\pi \times 41.66 \times 10^{-3} \times 800}{60}$

$$= 1.745 \text{ m/s}$$

✓ ***To find ψ_y :*** $\psi_y = \dfrac{b}{d_{1av}} = \dfrac{18.63}{41.66} = 0.447$

11. *IS quality 6 bevel gear* is assumed, from Table 5.22.

12. *Revision of design torque [M_t] :*

We know that, $[M_t] = M_t \times K \times K_d$ 8.15

where 8.15

$$K = 1.1, \quad \text{for } b/d_{1av} \leq 1, \text{ from Table 7.2, and}$$

$$K_d = 1.35, \quad \text{for IS quality 6 and } v \text{ upto 3 m/s, from Table 5.12.}$$

$$\therefore \quad [M_t] = 22.38 \times 1.1 \times 1.35 = 33.24 \text{ N-m}$$

13. *Check for bending :* We know that the induced bending stress (4) the formula

8.13A 8.13A

$$\sigma_b = \frac{R\sqrt{i^2+1}\ [M_t]}{(R-0.5\,b)^2 \times b \times m_t \times y_{v1}}$$

where $y_{v1} \approx 0.408$, for $z_{v1} = 23$, from Table 5.13

$$\therefore \quad \sigma_b = \frac{55.9\sqrt{2^2+1} \times 33.24 \times 10^3}{(55.9 - 0.5 \times 18.63)^2 \times 18.63 \times 2.5 \times 0.408}$$

$$= 100.75 \text{ N/mm}^2$$

We find $\sigma_b > [\sigma_b]$. Thus the design is not satisfactory.

Trial 2 : Now we will try with increased transverse module 3 mm. Repeating from Step again, we get

$$R = 0.5 \times m_t \times \sqrt{z_1^2 + z_2^2} = 0.5 \times 3 \times \sqrt{20^2 + 40^2} = 67.08 \text{ mm}$$

$$b = \frac{R}{\psi_y} = \frac{67.08}{3} = 22.36 \text{ mm}$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 3 - \frac{22.36 \times \sin 26.57°}{20} = 2.5 \text{ mm}$$

$$d_{1av} = m_{av} \times z_1 = 2.5 \times 20 = 50 \text{ mm}$$

8.15

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 50 \times 10^{-3} \times 800}{60} = 2.094 \text{ m/s}$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{22.36}{50} = 0.447$$

IS quality 6 bevel gear is assumed.

$$K = 1.1, \text{ from Table 7.2}$$

$$K_d = 1.35, \text{ from Table 5.12}$$

8.15

$$[M_t] = M_t \times K \times K_d = 22.38 \times 1.1 \times 1.35$$

$$= 33.24 \text{ N-m}$$

$$\sigma_b = \frac{67.08\sqrt{2^2+1} \times 33.24 \times 10^3}{(67.08 - 0.5 \times 22.36)^2 \times 22.36 \times 3 \times 0.408} = 58.3 \text{ N/mm}^2$$

Now we find $\sigma_b < [\sigma_b]$. Thus ***the design is satisfactory.***

14. ***Check for wear strength :*** We know that the induced contact stress,

2 for Checking Bevel 8.13

8.13 $$\sigma_c = \frac{0.72}{(R - 0.5\,b)}\left[\frac{\sqrt{(i^2+1)^3}}{i \times b} \times E_{eq}\,[M_t]\right]^{\frac{1}{2}}$$

8.13
$$= \frac{0.72}{(67.08 - 0.5 \times 22.36)}\left[\frac{\sqrt{(2^2+1)^3}}{2 \times 22.36} \times 1.4 \times 10^5 \times 33.24 \times 10^3\right]^{\frac{1}{2}}$$

$$= 439.33\ \text{N/mm}^2$$

We find $\sigma_c < [\sigma_c]$. Thus ***the design is satisfactory.***

15. ***Calculation of basic dimensions of pinion and gear :*** Refer Table 7.1. 8.38 Bevel

- ✓ Transverse module : m_t = **3 mm**
- ✓ Number of teeth : z_1 = **20** ; and z_2 = **40.**
- ✓ Pitch circle diameter : $d_1 = m_t \times z_1 = 3 \times 20 = \mathbf{60\ mm}$; and

8.38 $d_2 = m_t \times z_2 = 3 \times 40 = \mathbf{120\ mm.}$
- ✓ Cone distance : R = **67.08 mm**
- ✓ Face width : b = **22.36 mm**
- ✓ Pitch angles : δ_1 = **26.57°** ; and δ_2 = **63.43°**

8.38
- ✓ Tip diameter : $d_{a1} = m_t(z_1 + 2\cos\delta_1) = 3\,(20 + 2\cos 26.57°)$

  $= \mathbf{65.37\ mm}$; and

  $d_{a2} = m_t(z_2 + 2\cos\delta_2) = 3\,(40 + 2\cos 63.43°)$

  $= \mathbf{122.68\ mm}$
- ✓ Height factor : f_0 = **1**
- ✓ Clearance : c = **0.2**
- ✓ Addendum angle : $\tan\theta_{a1} = \tan\theta_{a2} = \dfrac{m_t \times f_0}{R} = \dfrac{3 \times 1}{67.08} = 0.0447$

  or $\theta_{a1} = \theta_{a2} = \mathbf{2.56°}$
- ✓ Dedendum angle : $\tan\theta_{f1} = \tan\theta_{f2} = \dfrac{m_t(f_0 + c)}{R} = \dfrac{3\,(1+0.2)}{67.08}$

8.38 $= 0.05366$

  or $\theta_{f1} = \theta_{f2} = \mathbf{3.07°}$
- ✓ Tip angle : $\delta_{a1} = \delta_1 + \theta_{a1} = 26.57° + 2.56° = \mathbf{29.13°}$; and

8.39 $\delta_{a2} = \delta_2 + \theta_{a2} = 63.43° + 2.56° = \mathbf{65.99°}$

PSG 8.39 ✓ Root angle : $\delta_{f1} = \delta_1 - \theta_{f1} = 26.57° - 3.07° = \mathbf{23.5°}$; and
$\delta_{f2} = \delta_2 - \theta_{f2} = 63.43° - 3.07° = \mathbf{60.36°}$

✓ Virtual number of teeth : z_{v1} = **23** ; and z_{v2} = **90.**

(4) **Example 7.12** ***Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 r.p.m. and the speed of the gear wheel shaft is 120 r.p.m. Pinion is of steel and wheel of cast iron. Each gear is expected to work hours / day for 10 years. The drive transmits 9.37 kW.***

Given Data : $\theta = 90°$; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; $P = 9.37$ kW.

To find : Design the bevel gear drive.

☺ Solution : Since the materials of pinion and gear are different, we have to design the ***pinion first and check the gear.***

1. ***Gear ratio :*** $i = \dfrac{N_1}{N_2} = \dfrac{360}{120} = 3$

Pitch angles : $\tan \delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

2. ***Material selection :*** Pinion – C45 Steel, $\sigma_u = 700$ N/mm² and $\sigma_y = 360$ N/mm²

1.40 Gear – CI grade 35, $\sigma_u = 350$ N/mm², from Table 5.3.

3. ***Gear life*** in hours = (2 hours/day) × (365 days / year × 10 years) = 7300 hours

∴ Gear life in cycles, $N = 7300 \times 360 \times 60 = 15.768 \times 10^7$ cycles

4. ***Calculation of initial design torque $[M_t]$:***

We know that, 8.15 $[M_t] = M_t \times K \times K_d$

where $M_t = \dfrac{60 \times P}{2\pi N_1} = \dfrac{60 \times 9.37 \times 10^3}{2\pi \times 360} = 248.6$ N-m, and

$K \cdot K_d = 1.3$, initially assumed.

∴ $[M_t] = 248.6 \times 1.3 = 323.28$ N-m

5. ***Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:***

8.4✓ ***To find E_{eq} :*** $E_{eq} = 1.7 \times 10^5$ N/mm², from Table 5.20.

✓ ***To find $[\sigma_{b1}]$:*** We know that the design bending stress for pinion,

$$[\sigma_{b1}] = \frac{1.4\, K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}, \text{ for rotation in one direction}$$