PSG 8.39 ✓ Root angle : $\delta_{f1} = \delta_1 - \theta_{f1} = 26.57° - 3.07° = \mathbf{23.5°}$; and

$\delta_{f2} = \delta_2 - \theta_{f2} = 63.43° - 3.07° = \mathbf{60.36°}$

✓ Virtual number of teeth : z_{v1} = **23** ; and z_{v2} = **90.**

(4) **Example 7.12** ***Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 r.p.m. and the speed of the gear wheel shaft is 120 r.p.m. Pinion is of steel and wheel of cast iron. Each gear is expected to work hours / day for 10 years. The drive transmits 9.37 kW.***

Given Data : $\theta = 90°$; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; P = 9.37 kW.

To find : Design the bevel gear drive.

☺ Solution : Since the materials of pinion and gear are different, we have to design ***the pinion first and check the gear.***

1. ***Gear ratio :*** $i = \frac{N_1}{N_2} = \frac{360}{120} = 3$

   Pitch angles : $\tan \delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$

   Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

2. ***Material selection :*** Pinion – C45 Steel, $\sigma_u = 700$ N/mm² and $\sigma_y = 360$ N/mm²

   1.40 Gear – CI grade 35, $\sigma_u = 350$ N/mm², from Table 5.3.

3. ***Gear life*** in hours = (2 hours/day) × (365 days / year × 10 years) = 7300 hours

   ∴ Gear life in cycles, N = 7300 × 360 × 60 = 15.768 × 10⁷ cycles

4. ***Calculation of initial design torque [M_t] :***

We know that, 8.15 $[M_t] = M_t \times K \times K_d$

where $M_t = \frac{60 \times P}{2 \pi N_1} = \frac{60 \times 9.37 \times 10^3}{2 \pi \times 360} = 248.6$ N-m, and

$K \cdot K_d = 1.3$, initially assumed.

∴ $[M_t] = 248.6 \times 1.3 = 323.28$ N-m

5. ***Calculation of E_{eq}, [σ_b] and [σ_c] :***

8.4✓ ***To find E_{eq} :*** $E_{eq} = 1.7 \times 10^5$ N/mm², from Table 5.20.

✓ ***To find [σ_{b1}] :*** We know that the design bending stress for pinion,

$$[\sigma_{b1}] = \frac{1.4 K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1} ,$$ for rotation in one direction

where

8.19 K_{bl} = 1, for HB ≤ 350 and N ≥ 10^7, from Table 5.14,

8.19 K_σ = 1.5, for steel pinion, from Table 5.15,

8.19 n = 2.5, steel hardened, from Table 5.17.

8.19 σ_{-1} = $0.25\,(\sigma_u + \sigma_y) + 50$, for forged steel, from Table 5.16.

$$= 0.25\,(700 + 360) + 50 = 315 \text{ N/mm}^2$$

$$\therefore \quad [\sigma_{bl}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 \text{ N/mm}^2$$

✓ ***To find*** $[\sigma_{cl}]$: We know that the design contact stress for pinion,

8.16 $$[\sigma_{cl}] = C_R \cdot HRC \times K_{cl}$$

where

8.16 C_R = 23, from Table 5.18,

8.16 HRC = 40 to 55, from Table 5.18, and

8.17 K_{cl} = 1, for steel pinion, HB ≤ 350 and N ≥ 10^7, from Table 5.19.

$$\therefore \quad [\sigma_{cl}] = 23 \times 50 \times 1 = 1150 \text{ N/mm}^2$$

6. *Calculation of cone distance (R) :*

1st formula PSG

We know that, $$R \geq \psi_y \sqrt{i^2 + 1}\; \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)\,[\sigma_c]}\right]^2 \times \frac{E_{eq}\,[M_t]}{i}}$$ 8.13

where 8.15 $\psi_y = R / b = 3$, initially assumed.

$$\therefore \quad R \geq 3\sqrt{3^2 + 1}\; \sqrt[3]{\left[\frac{0.72}{(3 - 0.5)\,1150}\right]^2 \times \frac{1.7 \times 10^5 \times 323.28 \times 10^3}{3}}$$

$$\geq 99.36$$

or $$R = 100 \text{ mm.}$$

7. Assume $z_1 = 20$; Then $z_2 = i \times z_1 = 3 \times 20 = 60$

Virtual number of teeth : $$z_{v1} = \frac{z_1}{\cos \delta_1} = \frac{20}{\cos 18.44°} \approx 22 \text{ ; and}$$

$$z_{v2} = \frac{z_2}{\cos \delta_2} = \frac{60}{\cos 71.56°} \approx 190.$$

8. *Calculation of transverse module* (m_t) **:**

We know that, $$m_t = \frac{R}{0.5\sqrt{z_1^2 + z_2^2}}$$

$$= \frac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162 \text{ mm}$$

From Table 5.8, the nearest higher standard transverse module is 4 mm.

9. *Revision of cone distance (R)*

We know that, $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 4 \sqrt{20^2 + 60^2} = 126.49$ mm

10. *Calculation of b, m_{av}, d_{1av}, v and ψ_y :*

✓ Face width (b) : $b = \dfrac{R}{\psi_y} = \dfrac{126.49}{3} = 42.16$ mm

8.15

✓ Average module (m_{av}) : $m_{av} = m_t - \dfrac{b \sin \delta_1}{z_1} = 4 - \dfrac{42.16 \times \sin 18.44°}{20}$

$= 3.333$

✓ Average pcd of pinion (d_{1av}) : $d_{1av} = m_{av} \times z_1 = 3.333 \times 20 = 66.66$ mm

✓ Pitch line velocity (v) : $v = \dfrac{\pi \times d_{1av} \times N_1}{60} = \dfrac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256$ m/s

✓ $\psi_y = \dfrac{b}{d_{1av}} = \dfrac{42.16}{66.66} = 0.632$

11. *IS quality 6 bevel gear* is assumed, from Table 5.22.

12. *Revision of design torque $[M_t]$:*

We know that, $[M_t] = M_t \times K \times K_d$

where

8.15 $K = 1.1$, from Table 7.2, and

8.16 $K_d = 1.35$, from Table 5.12.

$\therefore \quad [M_t] = 248.6 \times 1.1 \times 1.35 = 369.28$ N-m

13. Check *for bending of pinion :* We know that the induced bending stress,

8.13A $$\sigma_{b1} = \frac{R\sqrt{i^2+1}\ [M_t]}{(R - 0.5\, b)^2 \times b \times m_t \times y_{v1}}$$ th formula

where

8.18 $y_{v1} = 0.402$, for $z_{v1} = 22$, from Table 5.13

$$\therefore \quad \sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49 - 0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402} = 196.09 \text{ N/mm}^2$$

We find $\sigma_{b1} > [\sigma_{b1}]$. Thus the design is unsatisfactory.

Trial 2 : Now we will try with increased transverse module 5 mm. Repeating from Step again, we get

$$R = 0.5 \times m_t \times \sqrt{z_1^2 + z_2^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11 \text{ mm}$$

$$b = \frac{R}{\psi_y} = \frac{158.11}{3} = 52.7 \text{ mm}$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 5 - \frac{52.7 \times \sin 18.44}{20} = 4.166 \text{ mm}$$

$$d_{1av} = m_{av} \times z_1 = 4.166 \times 20 = 83.33 \text{ mm}$$

8.15 $$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = \mathbf{1.57 \text{ m/s}}$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{52.7}{83.33} = 0.632$$

IS quality 6 bevel gear is assumed.

$K = 1.1$; $K_d = 1.35$.

$M_t = 248.6 \times 1.1 \times 1.35 = 369.28$ N-m

$$\therefore \quad \sigma_{b1} = \frac{158.11\sqrt{3^2+1} \times 369.28 \times 10^3}{(158.11 - 0.5 \times 52.7)^2 \times 52.7 \times 5 \times 0.402} = 100.4 \text{ N/mm}^2$$

Now we find $\sigma_{b1} < [\sigma_{b1}]$, thus ***the design is satisfactory.***

14. *Check for wearing of pinion :* We know that the induced contact stress,

$$\sigma_{c1} = \left(\frac{0.72}{R - 0.5\,b}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i\,b} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}}$$ 8.13 PSG

$$= \left[\frac{0.72}{158.11 - 0.5 \times 52.7}\right]\left[\frac{\sqrt{(3^2+1)^3}}{3 \times 52.7} \times 1.7 \times 10^5 \times 369.28 \times 10^3\right]^{\frac{1}{2}}$$

$$= 612.33 \text{ N/mm}^2$$

We find $\sigma_{c1} < [\sigma_{c1}]$. Thus ***the design is satisfactory for pinion.***

15. *Check for gear (i.e., wheel) : Gear material :* CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel, $N = \dfrac{N_{pinion}}{3} = \dfrac{15.768 \times 10^7}{3} = 5.256 \times 10^7$ cycles

✓ ***To find $[\sigma_{b2}]$:*** We know that the design bending stress for gear,

$$[\sigma_{b2}] = \frac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1}$$

where $K_{bl} = \sqrt[9]{\dfrac{10^7}{N}} = \sqrt[9]{\dfrac{10^7}{5.256 \times 10^7}} = 0.832$, from Table 5.14,

8.20

8.19 $K_\sigma = 1.2$, from Table 5.15,

8.19 $n = 2$, from Table 5.17.

$$\sigma_{-1} = 0.45\,\sigma_u = 0.45 \times 350 = 157.5 \text{ N/mm}^2$$

$$\therefore \quad [\sigma_{b2}] = \frac{1.4 \times 0.832}{2 \times 1.2} \times 157.5 = 76.44 \text{ N/mm}^2$$

✓ ***To find*** $[\sigma_{c2}]$ **:** We know that the design contact stress for gear,

$$[\sigma_{c2}] = C_B \times HB \times K_{cl}$$

where $C_B = 2.3$, from Table 5.18,

PSG 8.16

8.16 $HB = 200$ to 260, from Table 5.18, and

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256 \times 10^7}} = 0.758$$

$$\therefore \quad [\sigma_{c2}] = 2.3 \times 260 \times 0.758 = 453.284 \text{ N/mm}^2$$

(a) Check for bending of gear : The induced bending stress for gear can be calculated using the relation

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

where 8.18 $y_{v1} = 0.402$, for $z_{v1} = 22$, from Table 5.13, and

PSG 8.18 $y_{v2} \approx 0.520$, for $z_{v2} = 190$, from Table 5.13.

$$\therefore \quad 100.4 \times 0.402 = \sigma_{b2} \times 0.520$$

or $$\sigma_{b2} = 77.6 \text{ N/mm}^2$$

We find σ_{b2} is almost equal to $[\sigma_{b2}]$. Thus the design is okay and it can be accepted.

(b) Check for wearing of gear : Since the contact area is same,

$$\sigma_{c2} = \sigma_{c1} = 612.33 \text{ N/mm}^2$$

We find $\sigma_{c2} > [\sigma_{c2}]$. It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 360 BHN. Then we get

$$[\sigma_{b2}] = 2.3 \times 360 \times 0.758 = 627.62 \text{ N/mm}^2.$$

Now we find $\sigma_{b2} < [\sigma_{b2}]$, thus the ***design is safe and satisfactory.*** S.