WORM GEAR

$$a = [(z_2/q)+1] \sqrt[3]{\left[\frac{540}{(z_2/q)[\sigma_c]}\right]^2 \frac{[M_t]}{10}} \qquad \ldots (8.31)$$

8.21. DESIGN PROCEDURE

Using Basic Equation

WORM GEAR

1. Select the suitable combination of materials for worm and worm wheel, consulting Tables 8.3 and 8.9.

2. Calculate the initial design torque $[M_t]$. Use $[M_t] = M_t \times K \times K_d$. Initially assume $K \cdot K_d = 1$.

3. ***Selection of z_1 and z_2:***

   ✓ Select the number of starts of worm depending on the efficiency requirement, consulting Table 8.4.

   ✓ Then, $z_2 = i \times z_1$.

4. ***Selection of $[\sigma_b]$ and $[\sigma_c]$:***

Select the design bending stress and design contact stress of the worm wheel from Table 8.9 and 8.10 respectively. To select $[\sigma_c]$, initially take $v_s = 3$ m/s.

5. ***Calculate the centre distance (a)*** using the equation 8.31. Choose initially diameter factor, $q = 11$. q can vary from 8 to 13.

6. ***Calculate the axial module** (m_x)* using the relation $m_x = 2\,a / (q + z_x)$. Then, choose the nearest higher standard axial module from Table 5.8.

7. ***Revise centre distance** (a)* using the relation $a = 0.5\, m_x\,(q + z_2)$

8. ***Calculate** $d,\ v,\ \gamma$ **and** v_s **:***

✓ Pitch diameter (d) : $$d_1 = q \times m_x \quad \text{and} \quad d_2 = z_2 \times m_x$$

✓ Pitch line velocity (v) : $$v_1 = \frac{\pi\, d_1\, N_1}{60} \quad \text{and} \quad v_2 = \frac{\pi\, d_2\, N_2}{60}$$

✓ Lead angle (γ) : $$\gamma = \tan^{-1}\left(\frac{z_1}{q}\right)$$

✓ Sliding velocity : $$v_s = \frac{v_1}{\cos\gamma}$$

9. ***Recalculate the design contact stress** $[\sigma_c]$* for the actual v_s, using Table 8.10.

10. ***Revise** K, K_d **and** $[M_t]$* for the actual velocity of the worm wheel (v_2).

11. ***Check for bending :***

✓ Calculate the induced bending stress using the equation 8.29.

✓ Compare the induced bending stress with the design bending stress.

✓ If $\sigma_b \le [\sigma_b]$, then the design is satisfactory.

12. If $\sigma_b > [\sigma_b]$, then the design is not satisfactory. Then increase the axial module.

13. ***Check for wear strength :***

✓ Calculate the induced contact stress using the equation 8.30.

✓ Compare the induced contact stress with the design contact stress.

✓ If $\sigma_c \le [\sigma_c]$, then the design is safe and satisfactory.

14. ***Check for efficiency :***

✓ If $\eta_{calculated} \ge \eta_{desired}$, then the design is satisfactory.

✓ Otherwise increase the lead angle γ.

15. Calculate the power loss and the area required to dissipate the heat.

16. ***Calculate all the basic dimensions of the worm and worm wheel*** using the equations listed in Table 8.2.

Example 8.15 *A steel worm running at 240 r.p.m. receives 1.5 kW from its shaft. The speed reduction is 10 : 1. Design the drive so as to have an efficiency of 80%. Also determine the cooling area required, if the temperature rise is restricted to 45 °C. Take overall heat transfer coefficient as 10 W/m² °C.*

Given Data : $N_1 = 240$ r.p.m. ; $P = 1.5$ kW ; $i = 10$; $\eta_{desired} = 80\%$; $t_o - t_a = 45°C$; $K_t = 10$ W/m² °C.

To find :

1. Design the worm gear drive, and
2. The cooling area required (A).

Solution : $\boxed{N_2 = N_1 / i} = 240/10 = 24$ r.p.m.

1. ***Selection of material :*** Worm – Steel ... (Given)

Wheel – Bronze (sand cast), selected from Tables 8.3 and 8.9.

2. ***Calculation of initial design wheel torque $[M_t]$:***

We know that, $$\boxed{[M_t] = M_t \times K \times K_d}$$

where $$M_t = \text{Wheel torque} = \boxed{\frac{60 \times P}{2\pi N_2}} = \frac{60 \times 1.5 \times 10^3}{2\pi \times 24}$$

$$= 596.83 \text{ N-m}$$

$$\boxed{K \cdot K_d = 1,} \text{ assumed initially.}$$

$\therefore$ Design wheel torque, $[M_t] = 596.83 \times 1 = 596.83$ N-m

3. ***Selection of z_1 and z_2 :***

✓ For $\eta = 80\%$, $z_1 = 3$ or 4, from Table 8.4. Here $z_1 = 3$ is selected.

✓ Then $\boxed{z_2 = i \times z_1} = 10 \times 3 = 30$.

4. ***Selection of $[\sigma_b]$ and $[\sigma_c]$:***

✓ For bronze wheel, $\sigma_u < 390$ N/mm², $[\sigma_b] = 50$ N/mm² is selected, for rotation in one direction, from Table 8.9.

✓ From Table 8.10, $[\sigma_c] = 159$ N/mm² is selected, assuming $v_s = 3$ m/s.

5. ***Calculation of centre distance (a) :***

We know that, $$\boxed{a = [(z_2/q) + 1] \sqrt[3]{\left[\frac{540}{(z_2/q)[\sigma_c]}\right]^2 \frac{[M_t]}{10}}}$$

where $q = 11$, initially chosen.

$$\therefore \quad a = [(30/11) + 1] \sqrt[3]{\left[\frac{540}{(30/11)\,159}\right]^2 \times \frac{596.83 \times 10^3}{10}}$$

$$a = 168.6 \text{ mm}$$

6. ***Calculation of axial module (m_x) :***

$$m_x = \frac{2\,a}{(q+z_2)} = \frac{2 \times 168.6}{(11+30)} = 8.22 \text{ mm}$$

From Table 5.8, the nearest higher standard axial module is 10 mm.

7. ***Revision of centre distance (a) :***

$$a = 0.5\, m_x\,(q+z_2) = 0.5 \times 10\,(11+30) = 205 \text{ mm}$$

8. ***Calculation of d, v, γ and v_s :***

✓ Pitch diameters : $d_1 = q \times m_x = 11 \times 10 = 110$ mm ; and

$$d_2 = z_2 \times m_x = 30 \times 10 = 300 \text{ mm.}$$

✓ Pitch line velocity : $v_1 = \dfrac{\pi\, d_1\, N_1}{60} = \dfrac{\pi \times 110 \times 10^{-3} \times 240}{60} = 1.382$ m/s ; and

$$v_2 = \frac{\pi\, d_2\, N_2}{60} = \frac{\pi \times 300 \times 10^{-3} \times 24}{60} = 0.377 \text{ m/s.}$$

✓ Lead angle : $\gamma = \tan^{-1}\left(\dfrac{z_1}{q}\right) = \tan^{-1}\left(\dfrac{3}{11}\right) = 15.25°$

✓ Sliding velocity : $v_s = \dfrac{v_1}{\cos\gamma} = \dfrac{1.382}{\cos 15.25°} = 1.432$ m/s

9. ***Recalculation of design contact stress $[\sigma_c]$:***

For $v_s = 1.432$ m/s, $[\sigma_c] \approx 172$ N/mm², from Table 8.10.

10. ***Revision of $[M_t]$:***

✓ For $v_2 < 3$ m/s, $K_d = 1$

$$\therefore [M_t] = M_t \times K \cdot K_d = 596.83 \times 1 \times 1 = 596.83 \text{ N-m}$$

11. ***Check for bending :*** We know that the induced bending stress,

$$\sigma_b = \frac{1.9\,[M_t]}{m_x^3 \times q \times z_2 \times y_v}$$

where y_v = Form factor based on virtual number of teeth, from Table 5.13.

$$z_v = \frac{z}{\cos^3\gamma} = \frac{30}{\cos^3 15.25°} \approx 34$$

$\therefore$ $y_v \approx 0.452$, for $z_v = 34$, from Table 8.7.

Then, $\sigma_b = \dfrac{1.9 \times 596.83 \times 10^3}{(10)^3 \times 11 \times 30 \times 0.452} = 7.6$ N/mm²

We find $\sigma_b < [\sigma_b]$, thus ***the design is satisfactory against bending.***

12. ***Check for wear :*** We know that the induced contact stress,

$$\sigma_c = \frac{540}{(z_2/q)}\sqrt{\left[\frac{(z_2/q)+1}{a}\right]^3 \times \frac{[M_t]}{10}}$$

$$= \frac{540}{(30/11)}\sqrt{\left[\frac{(30/11)+1}{205}\right]^3 \times \frac{596.83 \times 10^3}{10}}$$

$$= 118.59 \text{ N/mm}^2$$

We find $\sigma_c < [\sigma_c]$, thus ***the design is satisfactory against wear.***

13. ***Check for efficiency :***

$$\eta_{actual} = 0.95 \times \frac{\tan\gamma}{\tan(\gamma+\rho)}$$

where $\mu = 0.05$, for $v_s = 1.432$ m/s and bronze wheel, from graph 8.7.

$\therefore$ $\rho = \tan^{-1}(\mu) = \tan^{-1}(0.05) = 2.862°$

Then, $\eta_{actual} = 0.96 \times \dfrac{\tan 15.25°}{\tan(15.25° + 2.862°)} = 80\%$

We find $\eta_{actual} = \eta_{desired}$, thus ***the design is satisfactory.***

14. ***Calculation of cooling area required (A) :*** We know that,

$$(1-\eta) \times \text{Input power} = K_t \times A\,(t_o - t_a)$$

$$(1-0.8) \times 1.5 \times 10^3 = 10 \times A \times 45°$$

or Cooling area required, $A = 0.666 \text{ m}^2$

15. ***Calculation of basic dimensions of the worm and worm gear :*** Refer Table 8.2.

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- ✓ Axial module : m_x = **10 mm**
- ✓ Number of starts : z_1 = **3**
- ✓ Number of teeth on worm wheel : z_2 = **30**
- ✓ Length of worm : $L \geq (12.5 + 0.09 \times z_2)\, m_x$, from Table 8.5.

  $\geq (12.5 + 0.09 \times 30)\, 10$ = **152 mm**
- ✓ Centre distance : a = **205 mm**
- ✓ Face width : $b = 0.75\, d_1 = 0.75 \times 110$ = **82.5 mm**
- ✓ Height factor : f_0 = **1**
- ✓ Bottom clearance : $c = 0.25\, m_x = 0.25 \times 10$ = **2.5 mm**
- ✓ Pitch diameter : d_1 = **110 mm** ; and d_2 = **300 mm.**

✓ Tip diameter : $d_{a1} = d_1 + 2\,f_0 \cdot m_x = 110 + 2 \times 1 \times 10 = \mathbf{130\ mm}$; and

$d_{a2} = (z_2 + 2\,f_0)\,m_x = (30 + 2 \times 1)\,10 = \mathbf{320\ mm}$

✓ Root diameter : $d_{f1} = d_1 - 2\,f_0 \cdot m_x - 2 \times c$

$= 110 - 2 \times 1 \times 10 - 2 \times 2.5 = \mathbf{85\ mm}$; and

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PSG

$d_{f2} = (z_2 - 2\,f_0)\,m_x - 2 \times c$

$= (30 - 2 \times 1)\,10 - 2 \times 2.5 = \mathbf{275\ mm}$