DIFFERENTIATOR

One of the simplest of the op-amp circuits that contain capacitor is the differentiating amplifier, or differentiator. As the name suggests, the circuit performs the mathematical operation of differentiation, that is, the output waveform is the derivative of input waveform. A differentiator circuit is shown in Fig. 4.21 (a).

Analysis

The node N is at virtual ground potential i.e., $v_N = 0$. The current i_C through the capacitor is,

$$i_C = C_1 \frac{d}{dt}(v_i - v_N) = C_1 \frac{dv_i}{dt} \tag{4.68}$$

The current i_f through the feedback resistor is v_o/R_f and there is no current into the op-amp. Therefore, the nodal equation at node N is,

$$C_1 \frac{dv_i}{dt} + \frac{v_o}{R_f} = 0$$

from which we have

$$v_o = -R_f C_1 \frac{dv_i}{dt} \tag{4.69}$$

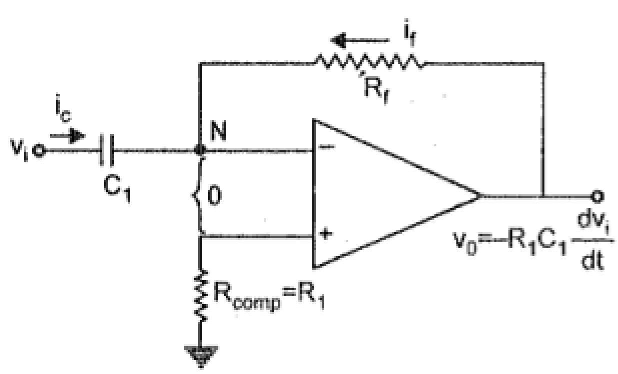

Fig. 4.21 (a) Op-amp differentiator

Thus the output voltage v_o is a constant $(-R_f C_1)$ times the derivative of the input voltage v_i and the circuit is a differentiator. The minus sign indicates a 180° phase shift of the output waveform v_o with respect to the input signal.

The phasor equivalent of Eq. (4.69) is, $V_o(s) = -R_f C_1\, s\, V_i(s)$ where V_o and V_i is the phasor representation of v_o and v_i. In steady state, put $s = j\omega$. We may now write the magnitude of gain A of the differentiator as,

$$|A| = \left|\frac{V_o}{V_i}\right| = \left|-j\omega R_f C_1\right| = \omega R_f C_1 \qquad (4.70)$$

From Eq. (4.70), one can draw the frequency response of the op-amp differentiator. Equation (4.70) may be rewritten as

$$|A| = \frac{f}{f_a}$$

where
$$f_a = \frac{1}{2\pi R_f C_1} \qquad (4.71)$$

At $f = f_a$, $|A| = 1$, i.e., 0 dB, and the gain increases at a rate of +20 dB/decade. Thus at high frequency, a differentiator may become unstable and break into oscillation. There is one more problem in the differentiator of Fig. 4.21 (a). The input impedance (i.e., $1/\omega C_1$) decreases with increase in frequency, thereby making the circuit sensitive to high frequency noise.

Practical Differentiator

A practical differentiator of the type shown in Fig. 4.21 (b) eliminates the problem of stability and high frequency noise.

The transfer function for the circuit in Fig. 4.21 (b) is given by,

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_f}{Z_i} = -\frac{s R_f C_1}{(1 + s R_f C_f)(1 + s C_1 R_1)} \qquad (4.72)$$

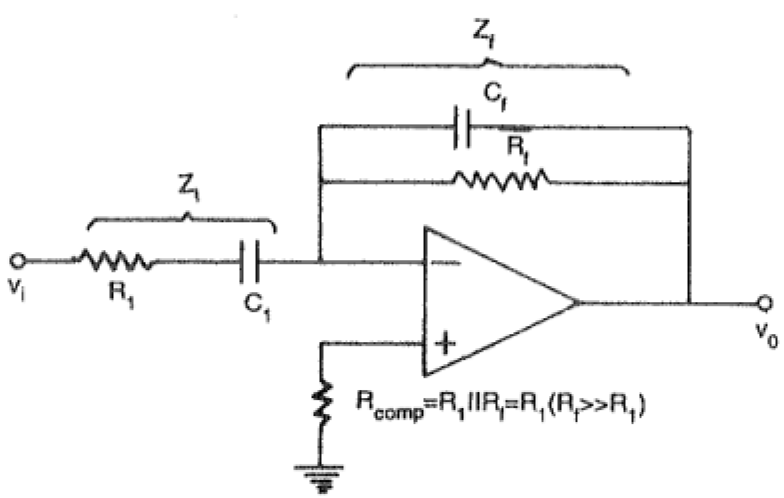

Fig. 4.21 (b) Practical differentiator

For $R_f C_f = R_1 C_1$, we get

$$\frac{V_0(s)}{V_i(s)} = -\frac{sR_fC_1}{(1+sR_1C_1)^2} = -\frac{sR_fC_1}{\left(1+j\frac{f}{f_b}\right)^2} \tag{4.73}$$

where,
$$f_b = \frac{1}{2\pi R_1 C_1} \tag{4.74}$$

From Eq. (4.73) it is evident that the gain increases at +20 dB/decade for frequency $f < f_b$ and decreases at –20 dB/decade for $f > f_b$ as shown by dashed lines in Fig. 4.21 (c). This 40 dB/decade change in gain is caused by $R_1 C_1$ and $R_f C_f$ factors. For the basic differentiator of Fig. 4.21 (a) the frequency response would have increased continuously at the rate of +20 dB/decade even beyond f_b causing stability problem at high frequency. Thus the gain at high frequency is reduced significantly, thereby avoiding the high frequency noise and stability problems. The value of f_b should be selected such that,

$$f_a < f_b < f_c$$

where f_c is the unity gain-bandwidth of the op-amp in open-loop configuration.

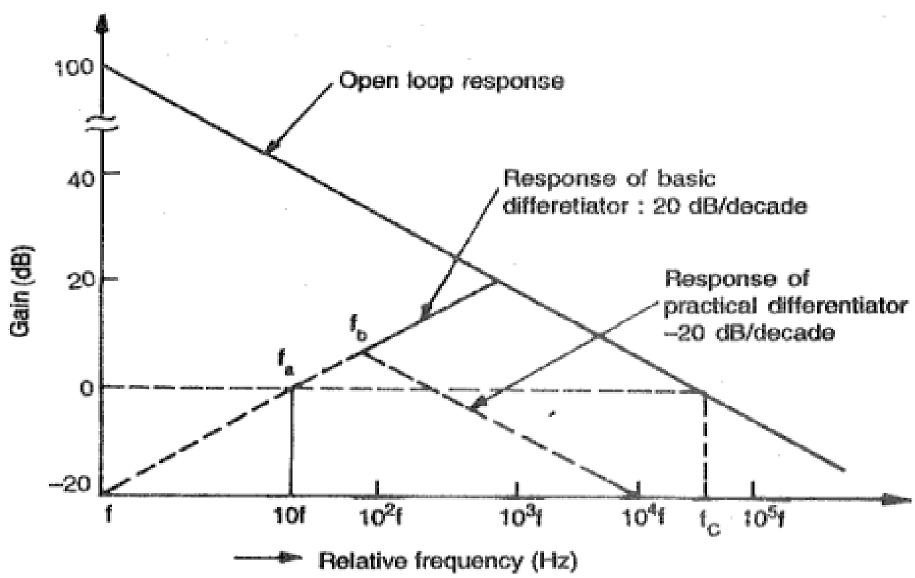

Fig. 4.21 (c) Frequency response

For good differentiation, one must ensure that the time period T of the input signal is larger than or equal to $R_f C_1$, that is,

$$T \geq R_f C_1 \tag{4.75}$$

It may be noted that for $R_f C_1$ much greater than $R_1 C_1$ or $R_f C_f$, Eq. (4.72) is reduced to, $V_o/V_i = -sR_f C_1$, that is, the expression of the output voltage remains the same as in the case of an ideal differentiator as

$$v_o = -R_f C_1 \frac{dv_i}{dt} \tag{4.76}$$

A resistance R_{comp} ($= R_1 \| R_f$) is normally connected to the (+) input terminal to compensate for the input bias circuit.

A good differentiator may be designed as per the following steps:

1. Choose f_a equal to the highest frequency of the input signal. Assume a practical value of C_1 (< 1μF) and then calculate R_f.
2. Choose $f_b = 10 f_a$ (say). Now calculate the values of R_1 and C_f so that $R_1 C_1 = R_f C_f$.

PROBLEM

(a) Design an op-amp differentiator that will differentiate an input signal with $f_{max} = 100$ Hz.
(b) Draw the output waveform for a sine wave of 1V peak at 100 Hz applied to the differentiator.
(c) Repeat part (b) for a square wave input.

Solution

(a) select, $f_a = f_{max} = 100 \text{ Hz} = \dfrac{1}{2\pi R_f C_1}$ [from Eq. (4.71)]

Let $C_1 = 0.1\ \mu\text{F}$,

then $R_f = \dfrac{1}{2\pi(10^2)(10^{-7})} = 15.9\ \text{k}\Omega$

Now choose

$$f_b = 10 f_a$$
$$= 1 \text{ kHz}$$
$$= \frac{1}{2\pi R_1 C_1} \quad \text{[from Eq. (4.74)]}$$

Therefore, $R_1 = \dfrac{1}{2\pi(10^3)(10^{-7})} = 1.59\ \text{k}\Omega$

Since $R_f C_f = R_1 C_1$,

we get, $C_f = \dfrac{1.59 \times 10^3 \times 10^{-7}}{15.9 \times 10^3} = 0.01\ \mu\text{F}$

(b) $v_i = 1 \sin 2\pi(100)t$

From Eq. (4.69),

$$v_o = -R_f C_1 \frac{dv_i}{dt}$$

$$= -(15.9\ \text{k}\Omega)\ (0.1\mu\text{F})\ \frac{d}{dt}[(1\ \text{V}) \sin (2\pi)(10^2)\ t]$$

$$= -(15.9\ \text{k}\Omega)\ (0.1\ \mu\text{F})\ (2\pi)\ (10^2)\ \cos [(2\pi)\ (10^2)t]$$

$$= -0.999 \cos [2\pi\ (10^2)t]$$

$$= -1 \cos [(2\pi)\ (10^2)\ t]$$

The input and output waveforms are shown in Fig. 4.22 (a).

(c) For a square wave input, say 1V peak and 1 KHz, the output waveform will consist of positive and negative spikes of magnitude V_{sat} which is approximately 13V for ± 15V op-amp power supply. During the time periods for which input is constant at ± 1V, the differentiated output will be zero. However, when input transits between ± 1V levels, the slope of the input is infinite for an ideal square wave. The output, therefore, gets clipped to about ± 13V for a ± 15V op-amp power supply as shown in Fig. 4.22 (b).

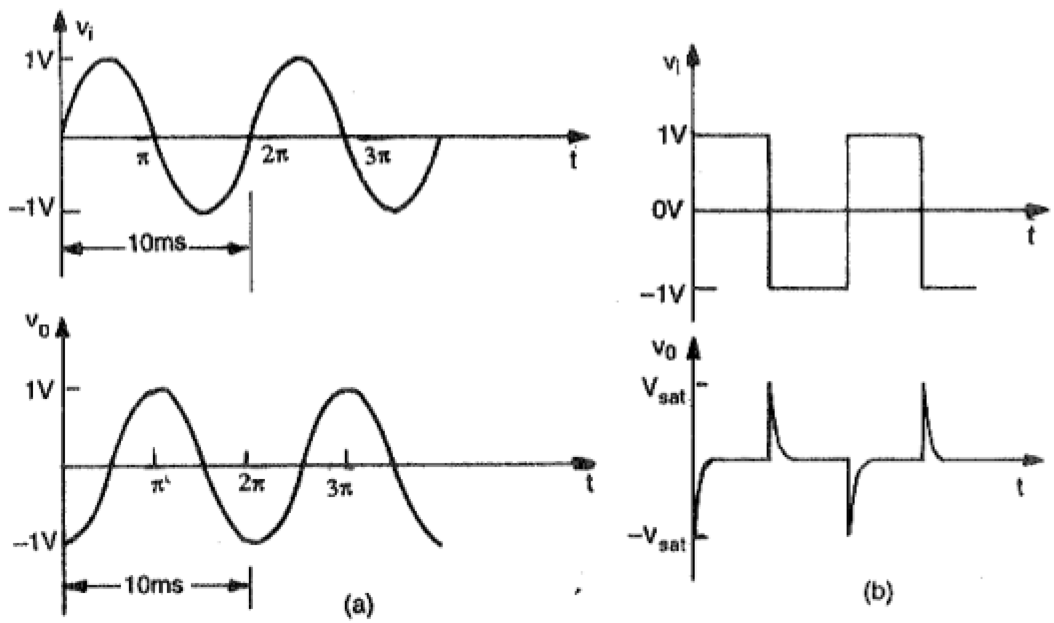

Fig. 4.22 (a) Sine-wave input and cosine output (b) Square wave input and spike output