INTEGRATOR

If we interchange the resistor and capacitor of the differentiator of Fig. 4.21 (a), we have the circuit of Fig. 4.23 (a) which as we will see, is an integrator. The nodal equation at node N is,

$$\frac{v_i}{R_1} + C_f \frac{dv_o}{dt} = 0 \qquad (4.77)$$

or,

$$\frac{dv_o}{dt} = -\frac{1}{R_1 C_f} v_i$$

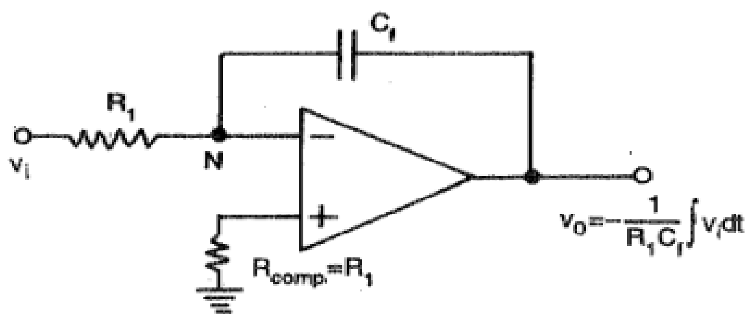

Fig. 4.23 (a) Op-amp integrator

Integrating both sides, we get,

$$\int_0^t dv_o = -\frac{1}{R_1 C_f} \int_0^t v_i \, dt$$

Practical Integrator Circuit (Lossy Integrator)

The gain of an integrator at low frequency (dc) can be limited to avoid the saturation problem if the feedback capacitor is shunted by a resistance R_f as shown in Fig. 4.23 (c). The parallel combination of R_f and C_f behaves like a practical capacitor which dissipates power unlike an ideal capacitor. For this reason, this circuit is also called a lossy integrator. The resistor R_f limits the low frequency gain to $-R_f/R_1$ (generally $R_f = 10\ R_1$) and thus provides dc stabilization.

Analysis

The nodal equation at the inverting input terminal of the op-amp of Fig. 4.23 (c) is,

$$\frac{V_i(s)}{R_1} + s\,C_f V_o(s) + \frac{V_o(s)}{R_f} = 0 \qquad (4.82)$$

from which we have,

$$V_o(s) = -\frac{1}{sR_1 C_f + R_1/R_f} V_i(s) \tag{4.83}$$

If R_f is large, the lossy integrator approximates the ideal integrator. For $s = j\omega$, magnitude of the gain of lossy integrator is given by

$$|A| = \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{\omega^2 R_1^2 C_f^2 + R_1^2/R_f^2}} = \frac{R_f/R_1}{\sqrt{1+(\omega R_f C_f)^2}} \tag{4.84}$$

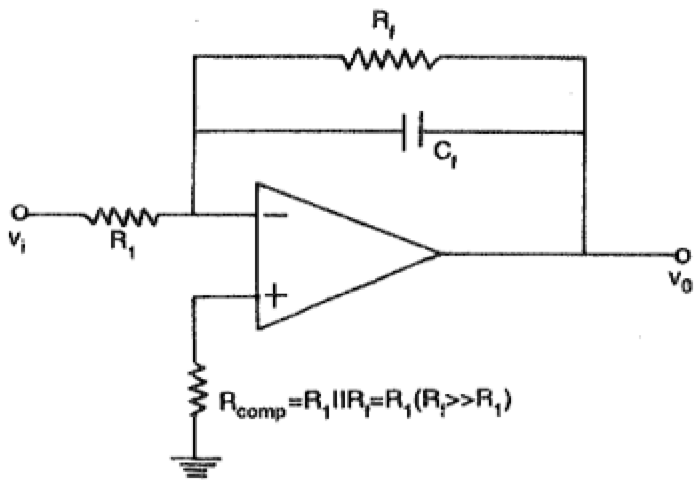

Fig. 4.23 (c) Practical or lossy integrator circuit

The Bode plot of the lossy integrator is also shown in Fig. 4.23 (b). At low frequencies gain is constant at R_f/R_1. The break frequency ($f = f_a$) at which the gain is 0.707 (R_f/R_1) (or – 3dB below its value of R_f/R_1) is calculated from Eq. (4.84) as

$$\sqrt{1+(\omega R_f C_f)^2} = \sqrt{2}$$

Solving for $f = f_a$, we get

$$f_a = \frac{1}{2\pi R_f C_f}$$

This is a very important frequency. It tells us where the useful integration range starts. If the input frequency is lower than f_a the circuit acts like a simple inverting amplifier and no integration results. At input frequency equal to f_a, 50% accuracy results. The practical thumb rule is that if the input frequency is 10 times f_a, than 99% accuracy can result.