



**SNS COLLEGE OF ENGINEERING**

**Coimbatore-107**



**COURSE NAME: ANALYSIS OF ALGORITHM**

**II YEAR/ IV SEMESTER**

**UNIT – V**

**BACKTRACKING ALGORITHM**

**Topic**

**N-Queen's Problem**



## UNIT - V BACKTRACKING METHOD

\*. We search for set of solution (or) optimal solution that satisfy some constraints using Back track formulation.

Solution =  $(x_1, x_2 \dots x_n)$

$x_i$  is chosen from this set which is best.

Note:

Search is refined at each stage by eliminating certain possibilities.

### Exhaustive Search:

It is basic routine of search in which all possible solutions to the give problem are produced.

### State Space Tree:

In backtracking while solving a given problem, a tree is constructed based on choices made. Such a tree with all



possible solutions is called state space tree.

### N Queens problem:

Step 1

	1	2	3	4
place $Q_1$	$Q_1$			
$Q_1$ in $(1,1)$				
2				
3				
4				

4 Queen is Solvable  
It should meet in  
→ No Row  
→ No Column  
→ No Diagonal.

Step 2

	1	2	3	4
place $Q_2$ in 2nd	$Q_1$			
Col Hit $(2,1) \times$			$Q_2$	
Diag Hit $(2,2) \times$				
No Hit & place $Q_2$ in 3				
4				

Step 3: X

	1	2	3	4
$Q_1$	$Q_1$			
$Q_2$			$Q_2$	
$Q_3 \rightarrow$				
Cannot be placed				
Dead end; So Back				

Step 3:

	1	2	3	4
place $Q_3$ in	$Q_1$			
by checking if				$Q_2$
it hits diagonal,				
Row (or) column			$Q_3$	
$(3,2)$ is correct				
to place $Q_3$				
4				

← Place  $Q_3$  again.

Step 4: Place  $Q_4$

	1	2	3	4
check	$Q_1$			
$Q_4$ to place in				$Q_2$
Row 4. But			$Q_3$	
it hits in				
diag Row & Col. 4				
0 occurs				

$Q_4$  cannot be placed; dead; So Back track  
Relocate it.





Steps:

Now  $Q_1$  relocated to (1,2) & again check  $Q_2, Q_3$  &  $Q_4$  to place.

	$Q_1$		
			$Q_2$
$Q_3$			
		$Q_4$	

No crossing.

& so placed all without crossing each other

Placing :  $Q_1(1,2)$ ,  $Q_2(2,4)$   
 $Q_3(3,1)$  and  $Q_4(4,3)$   
solution can be obtained.

Algorithm:

Queen(n)

for  $col \leftarrow 1$  to  $n$  do  
{

if (place(row, col)) then  
{

board(row) column

if (row = n) then

print board(n)

else

Queen(row + 1, n)

}

}

2 Queens  $\rightarrow$

	$Q_1$	
		$Q_2$

problem  $\nexists$

3 Queens  
problem

	$Q_1$		
			$Q_2$

Both cannot be solved  $\leftarrow$

It is because it is either in row or col



Time complexity (i) Promising Nodes:

$$O(N!) \rightarrow$$

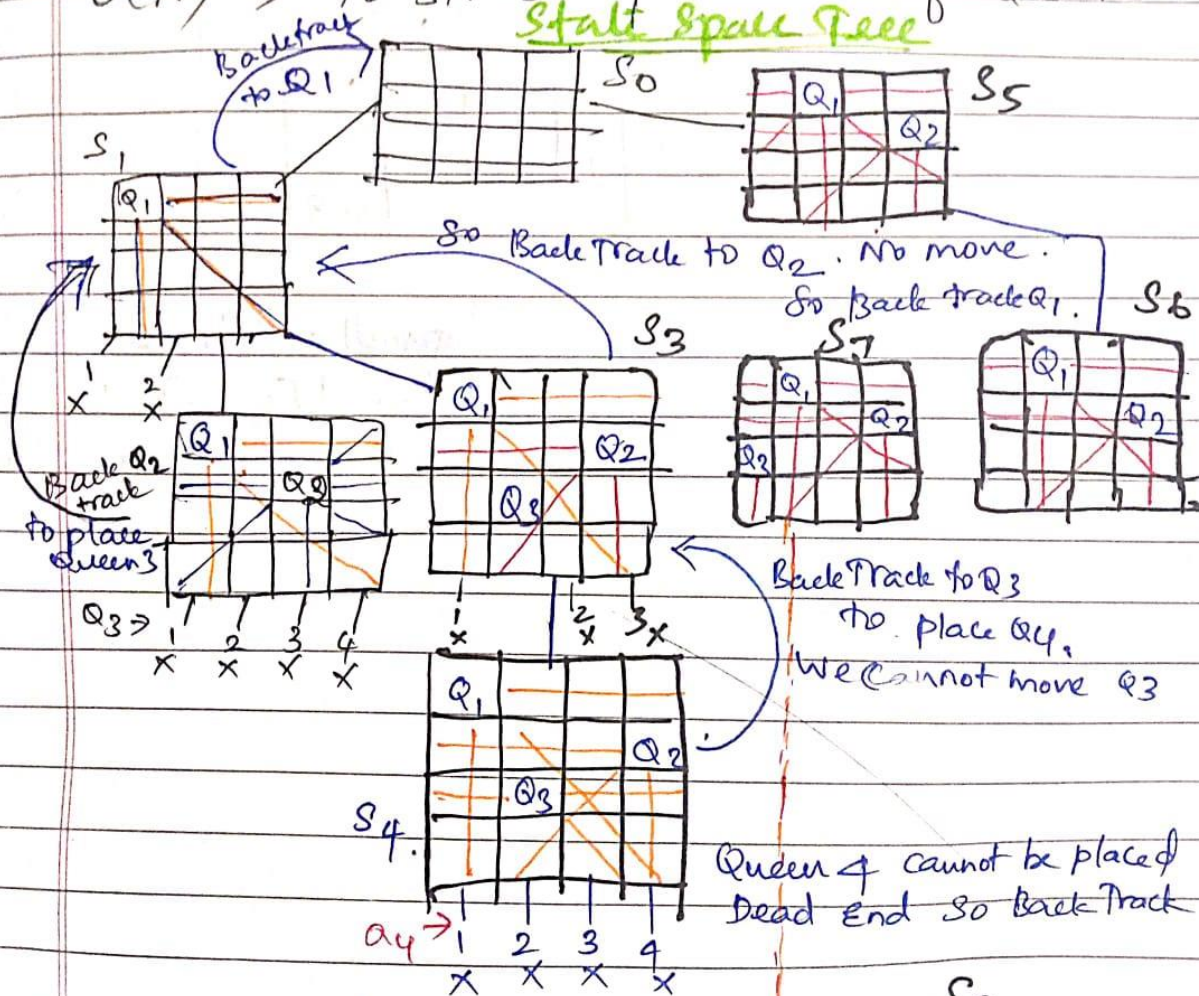
Lead to solution

(ii) Non promising Nodes:

Space complexity: Leads to no solution. (Dead End)

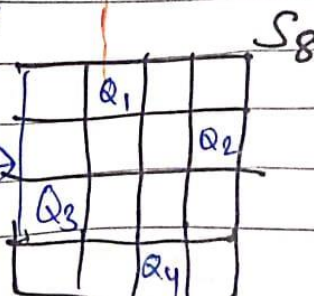
$O(N) \rightarrow$  To store positions of Queens

State Space Tree



one of  
Best  
feasible solution

$Q_1(1,2)$   
 $Q_2(2,4)$   
 $Q_3(3,1)$   
 $Q_4(4,3)$







## Algorithm & Analysis of N Queens

Algorithm solve N Queens (n)

```
{  
    board[n][n] = 0  
    placeQueen(board, row=0)  
    if (placeQueen returns true)  
        print ("Solution found")  
    else  
        Print (No solution);  
}
```

Time Complexity

$O(N!)$  → for each row; Try all possible

Space Complexity

$O(N)$  → single solution

$O(N \times K)$  → for storing all solutions

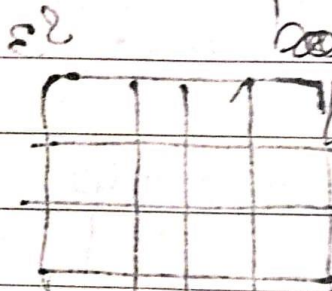
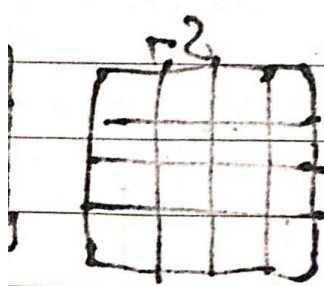
Algorithm placeQueen (board, row)

```
{  
    if (row > n)  
        return true // Queens placed successfully  
    for (col from 0 to n-1)  
        if (IsSafe (board, row, col)) == true  
            place queen at board [row][col] = 1  
            if placeQueen (board, row + 1) returns true  
                return true  
}
```



## 8 Queen's Solution.

	1	2	3	4	5	6	7	8
1	-	Q <sub>1</sub>	-	-	-	-	-	-
2	-	-	-	Q <sub>2</sub>	-	-	-	-
3	-	-	-	-	-	Q <sub>3</sub>	-	-
4	-	-	-	-	✓	-	-	Q <sub>4</sub>
5	-	-	-	-	Q <sub>5</sub>	-	-	-
6	Q <sub>6</sub>	-	-	-	-	-	-	-
7	-	Q <sub>7</sub>	-	-	-	-	-	-
8	-	-	-	-	-	Q <sub>8</sub>	-	-



board[4][4] = 0

board[0][0] = 1

isSafe(row, col)