



**SNS COLLEGE OF ENGINEERING**

**Coimbatore-107**



**COURSE NAME: ANALYSIS OF ALGORITHM**

**II YEAR/ IV SEMESTER**

**UNIT – V**

**BRANCH& BOUND ALGORITHM**

**Topic**

**Assignment Problem**



video : munigan  
Tech world

## Unit V Branch & Bound Technique

### Assignment problem

Branch & Bound Algorithm generates state space tree to obtain Best optimal value from current node of the decision tree. It is applicable to optimisation problem.

### Procedure for Assignment problems:

- ① specified by  $n \times n$  cost matrix
- ② Select 1 element from each row such that no two selected elements belong to same column.
- ③ calculate lower bound value by adding smallest element in each row.
- ④ select most promising node.
- ⑤ Generate children of promise nodes.
- ⑥ Last two steps are repeated until all the jobs are assigned to a person.

Note:

→ No jobs are assigned to two or more persons.

→ No persons can be assigned more



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1 Job = 1 person

Step 1: Find Lower Bound value.

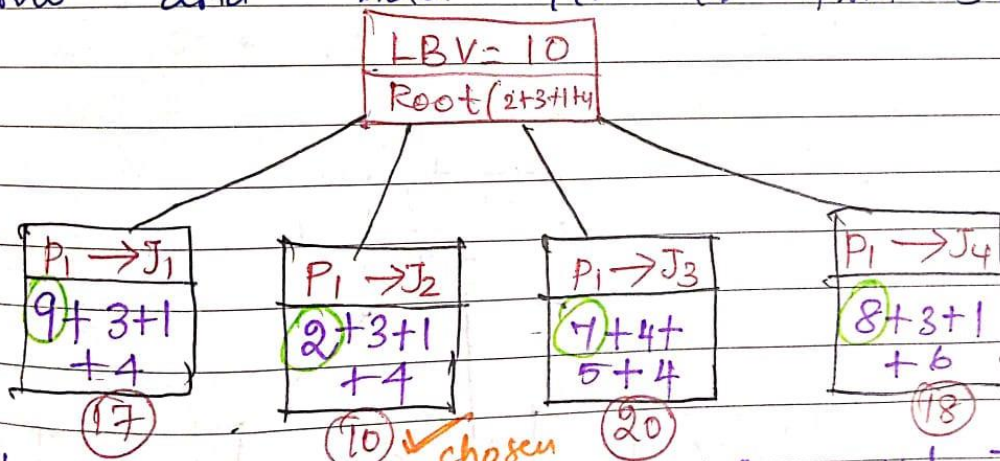
Example:

	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>
P <sub>1</sub>	9	2	7	8
P <sub>2</sub>	6	4	3	7
P <sub>3</sub>	5	8	1	8
P <sub>4</sub>	7	6	9	4

(LBV)

(i) Lower Bound Value =  $2 + 3 + 1 + 4 = 10$

Take smallest value from each row and add it to find LBV



Step 2: Assign P<sub>1</sub> with minimum value job

Only Constraint (X) ①. First value for all jobs is picked from table.

i.e.  $P_1 \rightarrow J_1$ : First value is 9 (J<sub>1</sub>);  $P_2 \rightarrow J_2$ : First value is 2 (from J<sub>2</sub>). Likewise  $P_3 \rightarrow J_3$ : 7,  $P_4 \rightarrow J_4$ : 4





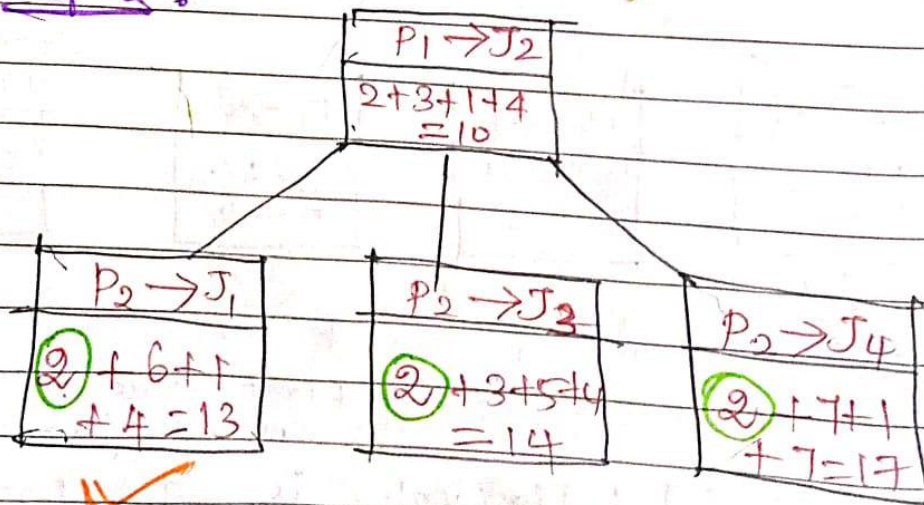
②. consider remaining values as  
least value from each row  
i.e.  $P_1 \rightarrow J_1$  : 3 from Row 2  
1 from Row 3  
4 from Row 4

Note: (X)

While choosing minimum element,  
consider that it should not be  
present in same column.

Eg:  $P_1 \rightarrow J_4$  :  $8+3+1+6$  is  
chosen. Instead of '4' we  
choose '6' because 4 (min value)  
is present in same 4th column

From Step 1 :  $P_1 \rightarrow J_2$  is assigned  
permanently because of minimum value  
Step 2:



✓  
Chosen 13 because it is min.



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	$J_1$	$J_2$	$J_3$	$J_4$
$P_1$	<del>9</del>	2	<del>4</del>	<del>8</del>
$P_2$	6	<del>4</del>	3	7
$P_3$	5	<del>8</del>	1	8
$P_4$	7	<del>6</del>	9	4

(i) Since  $P_1 \rightarrow J_2$  is allotted, Consider only 2 & omit remaining values of row 1.

(ii) For Creating  $P_2 \rightarrow J_1 : 2 + 6 + 1 + 4$   
 $P_2 \rightarrow J_1 : 13$   
 (2 is default value from Row 1; Select it  
 $P_2 \rightarrow J_1$  have 6 in row 2; select it.  
 Row 3 have min. value 1 & Row 4 have 4 as min. value; select it.)  
 — X — X — X —

(iii) Likewise  $P_2 \rightarrow J_3 : 2 + 3 + 5 + 4$ .  
 Select 2 (default);  $P_2 \rightarrow J_3 = 3$ ;  
 Row 3 have min value 1; But it cannot be chosen because it is in Column 3.  
 Row 4 include 4.  
 X — X — X — X — X —

(iv) Likewise  $P_2 \rightarrow J_4 : 2 + 7 + 1 + 6 = 17$ .  
 Select (i) 2 (default)  
 (ii)  $P_2 \rightarrow J_4 = 7$   
 (iii) 1 = min value in Row 3  
 (iv) ~~7~~  $\Rightarrow$  4 is min in Row 4. So select 4. But it is in Col 2. So choose 7  $\leftarrow$  Now 6 is min; But it is in Col 2.





Step 3:

$$\begin{array}{|l} P_2 \rightarrow J_1 \\ 2+6+1+4 \\ = 13 \end{array}$$

2, 6 should be written in 1st & 2nd row for previous answer.

$$\begin{array}{|l} P_3 \rightarrow J_3 \\ 2+6+1+4 \\ = 13 \end{array}$$

→ write value of  $P_3 \rightarrow J_3 = 1$  in 3rd term

$$\begin{array}{|l} P_3 \rightarrow J_4 \\ 2+6+8+9 \\ = 25 \end{array}$$

→ write value of  $P_3 \rightarrow J_4$  as 25th term

$P_3 \rightarrow J_4$  includes minimum value 18; & so it is assigned.

	$J_1$	$J_2$	$J_3$	$J_4$
$P_1$	9	2	4	8
$P_2$	6	14	3	7
$P_3$	5	8	1	8
$P_4$	7	6		

∴ By default 2, 6 from step 1 & 2 is included first in  $P_3 \rightarrow J_3$  and  $P_3 \rightarrow J_4$ .

$P_3 \rightarrow J_3$ ; Select min value; But cannot be taken because is present in column 3; So check value of  $P_3 \rightarrow J_3$



for  $P_3 \rightarrow J_4$ : (1) 2 & 6 from previous steps in Row 1 & Row 2 respectively.

(2) Row 3; minimum value '8' is chosen.

(3) Row 4 have min value '4' in 4<sup>th</sup> column; so choose '4'.

Note: Final solution is  $P_3 \rightarrow J_4$  (minimum) is chosen and it is assigned.

So we conclude with  $P_4$  is assigned with  $J_4$ .

Algorithm Branch Assign (level, current cost)

{ if (level == n) {

if (current cost < final cost)

final cost = current cost

return }

for (j = 0; j < n; j++)

{ if (assigned[j])

{ assigned[j] = 1;

temp = current cost + cost[level][j];

if (temp < final cost

Branch Assign (level + 1, temp);

assigned[j] = 0 }

Time complexity

For 'n' workers; there are n! possible assignments.

(worst case)

\* Best case (Pruning)

much less than n!

Space complexity:

Recursive version

$\rightarrow O(n)$

Priority Queue

$\rightarrow O(n^2)$ .