



SNS COLLEGE OF ENGINEERING

Coimbatore-107



COURSE NAME: ANALYSIS OF ALGORITHM

II YEAR/ IV SEMESTER

UNIT – V

BRANCH& BOUND ALGORITHM

Topic

Traveling Salesman Problem

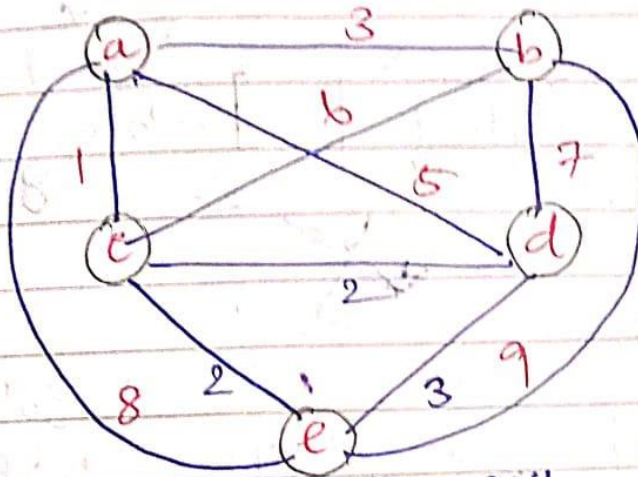


UNIT 5
Travelling Salesman Problem

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Example:

Logic \rightarrow Salesman traverse all the cities & come back to same city where he has started.



Step 1: Start from any vertex; check the neighbouring node it traverse. Select the two cities with minimum cost.

$$a \rightarrow ab + ac = 3 + 1 = 4$$

$$b \rightarrow ba + bc = 3 + 6 = 9$$

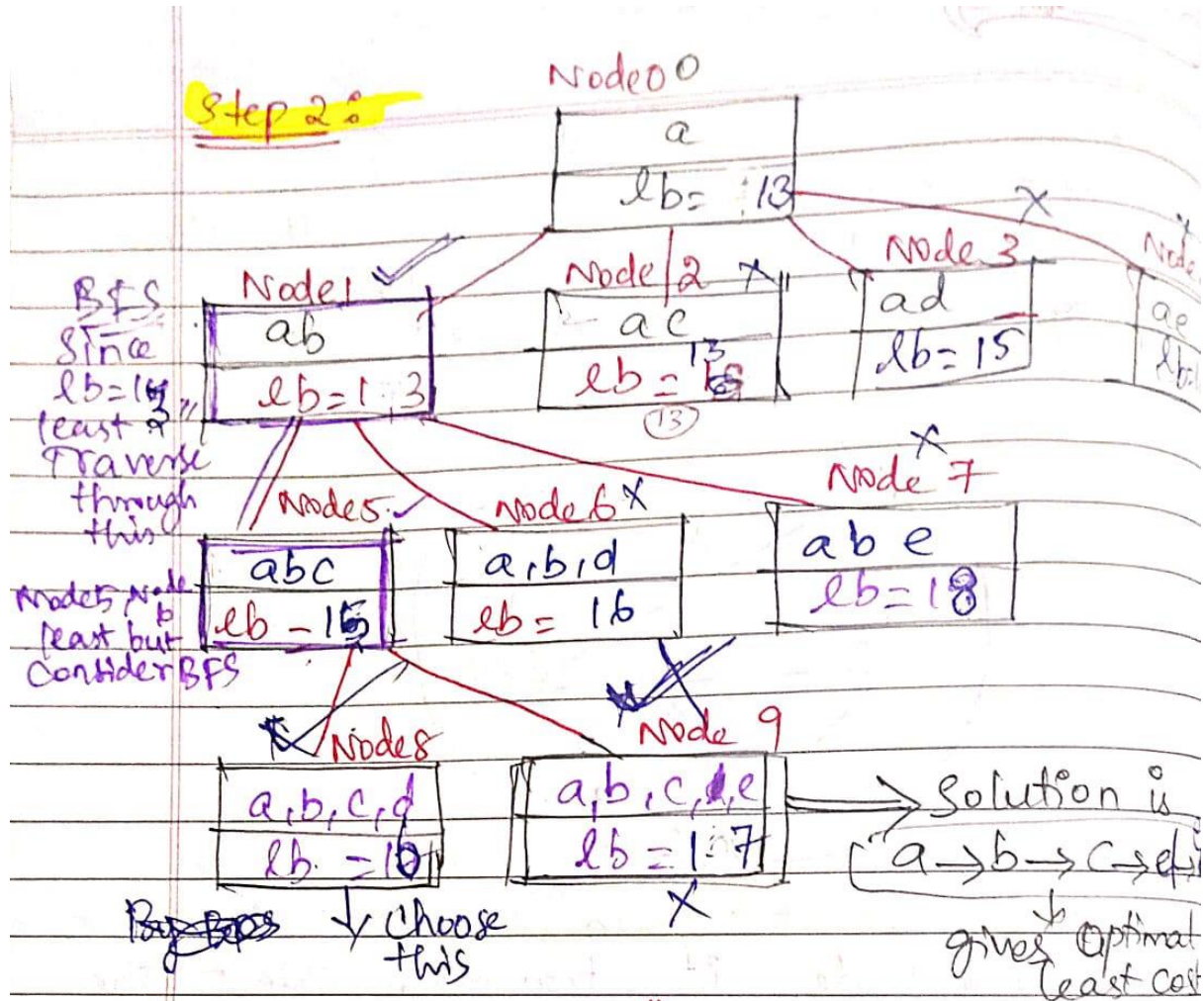
$$c \rightarrow ca + ce = 1 + 2 = 3$$

$$d \rightarrow dc + de = 2 + 3 = 5$$

$$e \rightarrow ec + ed = 2 + 3 = 5$$

26

$$lb = \left\lceil \frac{26}{2} \right\rceil = \frac{26}{2} = 13$$



Step 3: Node 1: "ab"

$$a \rightarrow ab + ac (\text{Next least}) = 3 + 1 = 4$$

$$b \rightarrow ab + bc (\text{Next least}) = 3 + 6 = 9$$

$$c \rightarrow 3 (\text{Previous Node value})$$

$$d \rightarrow 5 (\text{Previous Node value})$$

$$e \rightarrow 5 (\text{Previous Node value})$$

$$\underline{26}$$

$$lb = \left\lceil \frac{26}{2} \right\rceil = 13$$



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Step 4 : Node 2 : 'a, c' (least from a)

$$a \rightarrow ac + ab = 1 + 3 = 4$$

$$b \rightarrow 9$$

$$c \rightarrow ac + ce = 1 + 2 = 3$$

$$d \rightarrow 5$$

$$e \rightarrow 5$$

$$\frac{26}{2} = 13$$
$$lb = \frac{26}{2} = 13$$

Step 5 : Node 3 : 'a, d' (least from a)

$$a \rightarrow ad + ac = 5 + 1 = 6$$

$$b \rightarrow 9$$

$$c \rightarrow 3$$

$$d \rightarrow ad + de = 5 + 2 = 7$$

$$e \rightarrow 5$$

$$30$$

$$lb = \frac{30}{2} = 15$$

Step 6 : Node 4 : 'a, e' (least from a)

$$a \rightarrow ae + ac = 8 + 1 = 9$$

$$b \rightarrow 9$$

$$c \rightarrow 3$$

$$d \rightarrow 5$$

$$e \rightarrow ae + ec = 8 + 2 = 10$$

$$\frac{36}{2} = 18$$
$$lb = \frac{36}{2} = 18$$



Step 7: Node 5: "a, b, c" $\Rightarrow ab, bc$
 Since a is the first value $a \rightarrow ab + ac = 2 + 1 = 3$
 Since b is the first value $b \rightarrow ab + bc = 2 + 6 = 8$
 Since c is the first value $c \rightarrow bc + ac = 6 + 1 = 7$
 For defined values: $d \rightarrow 7$
 Take least from $e \rightarrow 5$
 Node 6: $lb = \frac{30}{2} = 15$
 For remaining just update from Node 5: $lb = 15$

Step 8: Node 6: "a, b, d" $\Rightarrow ab, bd$
 $a \rightarrow ab + ac = 3 + 1 = 4$
 $b \rightarrow ab + bd = 3 + 7 = 10$
 $c \rightarrow 3$
 $d \rightarrow bd + dc = 7 + 2 = 9$
 $e \rightarrow 5$
 $lb = \frac{31}{2} = 15.5$

$lb = \lceil \frac{31}{2} \rceil = \lceil 15.5 \rceil = 16$

Step 9: Node 7: "a, b, e" $\Rightarrow ab, be$
 $a \rightarrow ab + ac = 3 + 1 = 4$
 $b \rightarrow ab + be = 3 + 9 = 12$
 $c \rightarrow 3$
 $d \rightarrow 7.5$
 $e \rightarrow be + ec = 9 + 2 = 11$
 $lb = \frac{39}{2} = 19.5$



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$$lb = \left\lceil \frac{3+5}{2} \right\rceil = \lceil 1.7.5 \rceil = 18$$

$$lb = 18$$

Step 9: Node 8: a, b, c, d \Rightarrow ab, bc, cd

Minimum Cost \rightarrow

$$\begin{aligned} a &\rightarrow ab + ac = 3 + 1 = 4 \\ b &\rightarrow ab + bc = 3 + 6 = 9 \\ c &\rightarrow bc + cd = 6 + 2 = 8 \\ d &\rightarrow cd + de = 2 + 3 = 5 \\ e &\rightarrow 5 \end{aligned}$$

$$lb = \left\lceil \frac{3+1}{2} \right\rceil = 15.5 = 16$$

Node 9:

Step 10: a, b, c, d, e \Rightarrow ab, bc, ce

X

$$\begin{aligned} a &\Rightarrow ab + ac = 3 + 1 = 4 \\ b &\Rightarrow ab + bc = 3 + 6 = 9 \\ c &\Rightarrow bc + ce = 6 + 2 = 8 \\ d &\rightarrow 7 = 7 \\ e &\rightarrow ce + de = 2 + 3 = 5 \end{aligned}$$

$$lb = \left\lceil \frac{3+2}{2} \right\rceil = 16.5 = 17$$

Final Solution = a-b-c-d \rightarrow e \rightarrow g

This have minimum cost



Algorithm

Branch & Bound
(Travelling Salesman)

```
bound(currpath, n, graph, pos) // calculate bound
if (pos == n) // Function to perform Branch & Bound
{ // All cities visited, complete tour & Reset to 0
  if (graph[currpath[pos-1]][currpath[0]] != 0)
  { currcost += graph[currpath[pos-1]][currpath[0]]
    bestcost = min(bestcost, currcost)
  }
}
for (i = 0; i < n; i++) // Loop all city
{ if (not visited[i]) && graph[currpath[pos-1]][i] != 0
  & visited[i] = true
  currpath[pos] = i // calculate bound for curr
  boundcost = currcost + graph[currpath[pos-1]][i]
  Bound(currpath, n, graph, pos)
  if (boundcost < bestcost)
  { Branch & Bound(currpath, visited, n, graph,
    pos+1, currcost + graph[currpath[pos-1]][i])
  }
  visited[i] = false } }
```

Time complexity:

Without pruning:

$O(n!)$: Since it explores all possibilities of the cities to find optimal solution with pruning: Better than $O(n!)$.

Space complexity:

$O(n) \rightarrow$ depending no. of cities